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CALCULATION OF MULTISPAN FRAME STABILITY WITH REGARD TO GEOMETRIC NONLINEARITY

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Abstract: The paper considers the methodology of determining the critical value of the load acting on a flat multi-span frame, taking into account geometric nonlinearity. The elements of the frame have arbitrary stiffnesses constant along the length. The problem is solved in two stages. At the first stage, the strain calculation of the frame is performed, the main goal of which is to obtain the values of longitudinal forces in the struts. At the second stage, loss of stability of the first kind is considered in relation to the longitudinal bending of the props under the action of vertical forces. In both calculations, practically the same system of nonlinear equations of displacement method is solved. The compact notation of generalized stiffness matrix coefficients obtained earlier by the authors facilitates the development of the algorithm and computer programs designed for solving the problems set in the paper. The algorithm is implemented in Excel spreadsheets. To verify the obtained results, test calculations for both stages of the calculation have been performed. Using the proposed methodology, the stability of a flat free one-story frame with a periodic structure has been calculated. In the ANSYS software package, the calculations of this frame according to the deformed scheme were performed, with the subsequent determination of the value of critical longitudinal force in the struts. Comparison of the results of calculations by the offered technique and in the ANSYS program complex shows their practically complete coincidence: the difference in the values of longitudinal forces in the frame struts with allowance for geometric nonlinearity is less than 0.01 %; the calculation results of the first two critical forces differ by 0.06 %. The proposed methodology allows us to use a unified approach to the formation of systems of solving nonlinear equations, both in the strain calculation and in the calculation of stability. In addition, this approach releases from the use of expensive computer programs, the use of which requires special training.

Keywords: strain calculation, rod stability, displacement method, multi-span frame, stability equation, critical longitudinal forces, Excel, ANSYS.

РОЗРАХУНОК БАГАТОПРОЛЬОТНОЇ РАМИ НА СТІЙКІСТЬ З УРАХУВАННЯМ ГЕОМЕТРИЧНОЇ НЕЛІНІЙНОСТІ

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Анотація: У статті розглядається методика визначення критичного значення навантаження, що діє на плоску багатопрогонову раму з урахуванням геометричної нелінійності. Елементи рами мають довільні жорсткості, постійні по довжині. Поставлене завдання вирішується у два етапи. На першому етапі виконується деформаційний розрахунок рами, основною метою якого є набуття значень поздовжніх зусиль у колонах. На другому етапі розглядається втрата стійкості першого роду стосовно поздовжнього вигину колон під дією вертикальних сил. В обох розрахунках вирішується практично однакова система нелінійних рівнянь способу переміщень. Раніше отриманий авторами компактний запис узагальнених коефіцієнтів матриці жорсткості суттєво полегшує розробку алгоритму та комп'ютерних програм, призначених для вирішення поставлених у роботі завдань. Реалізація алгоритму виконано у загальнодоступних таблицях Excel. Для верифікації отриманих результатів виконані тестові розрахунки обох етапів розрахунку. За запропонованою методикою виконано розрахунок на стійкість плоскої вільної шести прогонової одноповерхової симетричної рами. На першому етапі обчислень на раму діє система зосереджених вертикальних сил у вузлах та рівномірно розподілене навантаження у всіх прольотах. Внаслідок виконання деформаційного розрахунку визначено внутрішні зусилля в елементах моделі. При розрахунку стійкість на раму діє лише приведені вузлові вертикальні сили, значення яких пропорційні поздовжнім зусиллям у колонах, отриманим першому етапі. В результаті виконаного розрахунку визначено критичні значення поздовжніх зусиль у колонах, що відповідають двом першим формам втрати стійкості. У програмному комплексі ANSYS виконано розрахунки цієї рами за деформованою схемою, з подальшим визначенням величини критичного поздовжнього зусилля в колонах. Порівняння результатів розрахунків за запропонованою методикою та у програмному комплексі ANSYS показує їх практично повний збіг як у значеннях поздовжніх сил у колонах рами з урахуванням геометричної нелінійності, так і в результатах обчислень перших двох критичних сил. Запропонована методика дозволяє використовувати єдиний підхід до формування систем нелінійних рівнянь, як у деформаційному розрахунку, так і в розрахунку на стійкість.

Ключові слова: деформаційний розрахунок, стійкість стрижня, метод переміщень, багатопрогонова рама, рівняння стійкості, критичне поздовжнє зусилля, Excel, ANSYS.

1 INTRODUCTION

Widespread construction structures include multi-span frames of industrial and residential buildings. It is possible to distinguish several decisive factors determining the bearing capacity of such frames consisting of flexible compressed and compression-curved elements: possibility of stability loss of separate elements or of the whole structure; consideration of geometrical deformations of frame elements. It is a complicated mathematical problem of obtaining the exact solution of set tasks considering these factors for multi-span frames. There are several ways of its solution. In some cases, additional hypotheses are introduced, allowing to essentially simplify the frame calculation scheme and, as a consequence, to obtain an exact solution. In other cases, approximate calculation methods are used, which are implemented with the use of modern computing software complexes. However, for multi-span flat frames of a periodic structure subjected only to the vertical load, it is possible to combine these ways - without changing the frame calculation scheme, to obtain the exact solution of the task in view of the above factors.

2 ANALYSIS OF LITERATURE DATA AND PROBLEM STATEMENT

The issue of determining the critical load at loss of stability in multi-span frames with regard to nonlinearity is reflected in numerous works of domestic and foreign scientists.

Works [1-5] are devoted to development of methods of engineering calculation of spatial frames for stability taking into account physical and geometrical nonlinearities. The modern state of the problem of accounting geometric nonlinearity in structural models is considered in [6].

In a number of works to solve this problem, the classical method of displacements is used with subsequent determination of critical load from nonlinear equations. For this purpose, approximate methods are used in which geometric assumptions are introduced into the mathematical model in frame models [7-9]. Other works use the correction of model element stiffnesses [5, 10, 11]. In works [1, 2, 9, 12] and many others, the engineering methods of stability calculation are oriented to the use of modern computer software, although for flat frames of periodic structure it is not always economically feasible.

The closest to the problem considered in this article is the engineering method of calculation [12], which allowed to simplify the testing of frame rods for stability without using modern software computing complexes for this purpose. However, there are no proposals for the use of engineering methods to account for geometric nonlinearity in the calculation of multispan frames for stability in the literature.

3 PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of this work is to develop a technique for determining the critical value of the load acting on a flat multi-span frame, taking into account geometric nonlinearity.

4 RESEARCH RESULTS

Consider a flat multi-span frame of periodic structure under the action of a system of vertical loads (Fig. 1). In the general case, the elements of the frame have arbitrary geometrical and physical-mechanical characteristics constant along the length of the elements. On the example of such a frame, let's develop a technique for determining the critical value of the load, taking into account geometric nonlinearity. In order to achieve the set task, we shall sequentially perform two calculations - strain calculation of a flat multi-span frame under the action of a vertically placed combined load system and calculation of the stability of the same frame under the action of vertical forces. All calculations will be performed by means of displacement method.

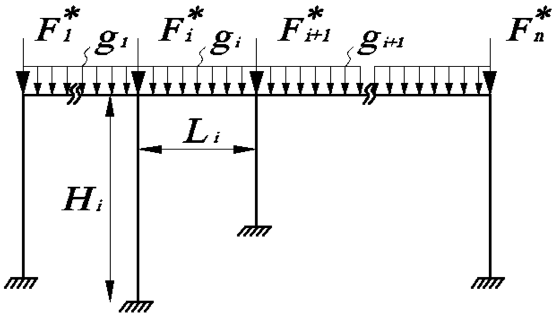

Fig. 1. Model of a flat multi-span frame

As a result of the first calculation, let's determine the longitudinal forces in vertical bars of the frame, with due account for geometric changes of the given model. Let's consider an elastic statement of the problem under the assumption of small deformations [14]. Let us perform the calculation step by step. At the first step, we define the internal forces in the frame according to the nondeformed scheme, and perform the subsequent calculation by the method of successive approximations, taking into account the values of longitudinal forces in the compressed rods of the frame obtained during the previous iterations.

Given that the unknowns of the displacement method are angular and linear displacements, this model of the frame has the number of unknowns of the displacement method equal to n+1, where n is the number of columns. The basic system of the displacement method of the model is shown in Fig. 2.

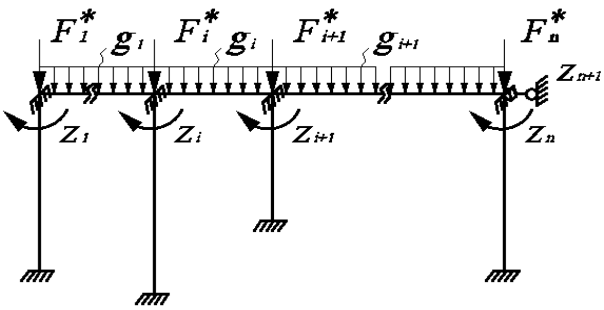

Fig. 2. The basic system of displacement method in strain calculations

The system of canonical equations of the displacement method in matrix form has the form:

$$\begin{vmatrix} r_{1,1}(\nu) & r_{1,2}(\nu) & \dots & r_{1,n}(\nu) & r_{1,n+1}(\nu) \\ r_{2,1}(\nu) & r_{2,2}(\nu) & \dots & r_{2,n}(\nu) & r_{2,n+1}(\nu) \\ \dots & \dots & \dots & \dots & \dots \\ r_{n,1}(\nu) & r_{n,2}(\nu) & \dots & r_{n,n}(\nu) & r_{n,n+1}(\nu) \\ r_{n+1,1}(\nu) & r_{n+1,2}(\nu) & \dots & r_{n+1,n}(\nu) & r_{n+1,n+1}(\nu) \end{vmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_n \\ Z_{n+1} \end{bmatrix} = - \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_n \\ R_{n+1} \end{bmatrix}. \tag{1}$$

Let us construct the unit diagrams necessary to determine the coefficients of the system of canonical equations. At their construction we will take into account the longitudinal bending from the action of the system of forces, and the influence of compressive forces will be taken into account by means of the corresponding nonlinear functions $\varphi_2(\nu_i)$, $\varphi_4(\nu_i)$, $\eta_2(\nu_i)$:

$$\varphi_2(\nu_i)=\frac{\nu_i\cdot\left(tg\nu_i-\nu_i\right)}{8tg\nu_i\cdot\left(tg\frac{\nu_i}{2}-\frac{\nu_i}{2}\right)};\quad \varphi_4(\nu_i)=\frac{\left(\frac{\nu_i}{2}\right)^2 tg\frac{\nu_i}{2}}{3\left(tg\frac{\nu_i}{2}-\frac{\nu_i}{2}\right)};\quad \eta_2(\nu_i)=\frac{\left(\frac{\nu_i}{2}\right)^3}{3\left(tg\frac{\nu_i}{2}-\frac{\nu_i}{2}\right)},$$

where $\nu_i=\alpha_i H_i=H_i\sqrt{F_i/EI_{Ci}}$.

For the considered frame model, we will represent the symmetric matrix of the coefficients of equation (1) with dimensions (n+1)×(n+1) in the block form:

$$\left\|\begin{matrix}\begin{vmatrix} r_{1,1}(\nu) & r_{1,2}(\nu) & \cdots\cdots & r_{1,n}(\nu)\\ r_{2,1}(\nu) & r_{2,2}(\nu) & \cdots\cdots & r_{2,n}(\nu)\\ \cdots & \cdots & \cdots\cdots & \cdots\\ r_{n,1}(\nu) & r_{n,2}(\nu) & \cdots & r_{n,n}(\nu)\end{vmatrix} & \begin{vmatrix} r_{1,n+1}(\nu)\\ r_{2,n+1}(\nu)\\ \cdots\\ r_{n,n+1}(\nu)\end{vmatrix}\\ \begin{vmatrix} r_{n+1,1}(\nu) & r_{n+1,2}(\nu) & \cdots & r_{n+1,n}(\nu)\end{vmatrix} & \begin{vmatrix} r_{n+1,n+1}(\nu)\end{vmatrix}\end{matrix}\right\|. \tag{2}$$

The physical meaning of the elements of blocks I (symmetric ribbon matrix of size n×n) and II (column of size n) are the reaction moments in the superimposed rigid links; the elements of blocks III (transposed block II) and VI (one element) are the reactions in the superimposed linear links.

Taking into account the introduced coefficients of variation of the model characteristics and (lengths of struts and beams); and (values of bending stiffness of struts and beams); and (values of concentrated forces and intensity of uniformly distributed load), let us define all its design characteristics:

$$\begin{aligned}&H_i=H_o h_i;\ L_i=L_o k_i;\\ &EI_{Ci}=EI_{Co}s_i;\ EI_{Pi}=EI_{Po}p_i;\\ &F_i=F_o f_i;\ \ q_i=q_o z_i\end{aligned} \tag{3}$$

Taking into account the symbols introduced for the constants $a=4EI_{Co}/H_o$; $b=4EI_{Po}/L_o$; $c=12EI_{Co}/H_o^3$; $d=-6EI_{Co}/H_o^2$, formulas of 4 generalized coefficients of the specified blocks [13], we obtain a compact notation of any of the elements of the block matrices of the general equation (1):

$$\begin{aligned}&r_{i,i}=a\cdot\frac{s_i}{h_i}\varphi_2(\nu_i)+b\cdot\frac{p_{i-1}}{k_{i-1}}+b\cdot\frac{p_i}{k_i};\ r_{i,i-1}=\frac{b}{2}\cdot\frac{p_i}{k_i};\\ &r_{n+1,i}=-\frac{6EI_{Ci}}{H_i^2}\varphi_4(\nu_i)=d\cdot\frac{s_i}{h_i^2}\varphi_4(\nu_i);\\ &r_{n+1,n+1}=c\cdot\left[\frac{s_1}{h_1^3}\eta_2(\nu_1)+\frac{s_2}{h_2^3}\eta_2(\nu_2)+\ldots+\frac{s_n}{h_n^3}\eta_2(\nu_n)\right].\end{aligned} \tag{4}$$

At the first step of the deformation calculation in the system of canonical equations (1) all nonlinear functions $\varphi_2(\nu_i)$, $\varphi_4(\nu_i)$, $\eta_2(\nu_i)$ are taken as singular (5). We find the displacements of nodes in the model in the elastic formulation of the problem, followed by determination of all internal forces in the rods.

$$\begin{vmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,n} & r_{1,n+1} \\ r_{2,1} & r_{2,2} & \dots & r_{2,n} & r_{2,n+1} \\ \dots & \dots & \dots & \dots & \dots \\ r_{n,1} & r_{n,2} & \dots & r_{n,n} & r_{n,n+1} \\ r_{n+1,1} & r_{n+1,2} & \dots & r_{n+1,n} & r_{n+1,n+1} \end{vmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_n \\ Z_{n+1} \end{bmatrix} = - \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_n \\ R_{n+1} \end{bmatrix}. \quad (5)$$

On the second and subsequent iterations of the strain calculation, the influence of the compressive forces obtained earlier for each rod of the model is taken into account by calculating (correcting) the corresponding nonlinear functions of longitudinal bending $\varphi_2(\nu_i)$, $\varphi_4(\nu_i)$, $\eta_2(\nu_i)$. We determine anew the values of all coefficients (4) and solve the system of equations (1). The number of such iterations depends on the convergence of the computational process. As the criterion of convergence, we choose the value of maximum difference in longitudinal forces of each rod in the last two iterations of the calculation.

Now, let's proceed to the second calculation, the goal of which is to determine the critical forces in the problem of loss of stability of the first kind as applied to the longitudinal bending of columns. In contrast to the previously considered calculation scheme, in the second calculation we will consider only the system of vertical concentrated forces in the nodes. The values of these forces are obtained in the deformation calculation and are equal to the values of longitudinal forces in the columns. The basic system of displacement method in the stability analysis is illustrated in Fig. 3.

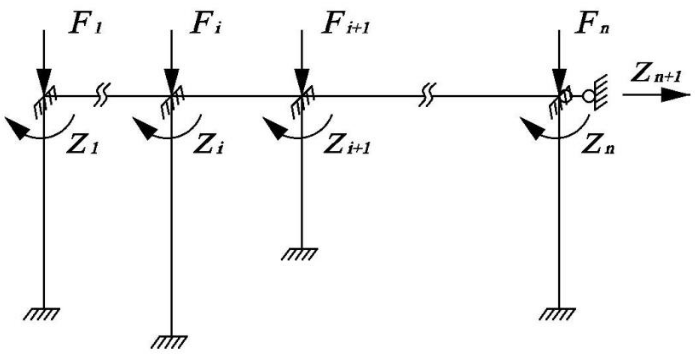

Fig. 3. Basic system of displacement analysis in stability analysis

Let us connect the vertical forces equal in magnitude to the longitudinal forces, obtained in the deformation analysis. They will change in proportion to a single parameter F_o:

$$F_1 = f_1 \cdot F_o; \dots F_2 = f_2 \cdot F_o; \dots, F_n = f_n \cdot F_o. \quad (6)$$

We shall equate the weight coefficients of the system of equations (1) to zero due to the nodal application of the load:

$$\begin{vmatrix} r_{1,1}(\nu) & r_{1,2}(\nu) & \dots & r_{1,n}(\nu) & r_{1,n+1}(\nu) \\ r_{2,1}(\nu) & r_{2,2}(\nu) & \dots & r_{2,n}(\nu) & r_{2,n+1}(\nu) \\ \dots & \dots & \dots & \dots & \dots \\ r_{n,1}(\nu) & r_{n,2}(\nu) & \dots & r_{n,n}(\nu) & r_{n,n+1}(\nu) \\ r_{n+1,1}(\nu) & r_{n+1,2}(\nu) & \dots & r_{n+1,n}(\nu) & r_{n+1,n+1}(\nu) \end{vmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_n \\ Z_{n+1} \end{bmatrix} = 0. \quad (7)$$

The search for critical forces consists in determining the roots of the general stability equation, which we will obtain by equating the determinant of the coefficient matrix of the system (7) to zero, provided that the displacements $Z_i \neq 0$:

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$$D(\nu)=\begin{vmatrix} r_{1,1}(\nu) & r_{1,2}(\nu) & \dots & r_{1,n}(\nu) & r_{1,n+1}(\nu) \\ r_{2,1}(\nu) & r_{2,2}(\nu) & \dots & r_{2,n}(\nu) & r_{2,n+1}(\nu) \\ \dots & \dots & \dots & \dots & \dots \\ r_{n,1}(\nu) & r_{n,2}(\nu) & \dots & r_{n,n}(\nu) & r_{n,n+1}(\nu) \\ r_{n+1,1}(\nu) & r_{n+1,2}(\nu) & \dots & r_{n+1,n}(\nu) & r_{n+1,n+1}(\nu) \end{vmatrix}=0. \quad (8)$$

All nonlinear parameters ν_i of functions $\varphi_2(\nu_i)$, $\varphi_4(\nu_i)$, $\eta_2(\nu_i)$, included in the general stability equation (8), are reduced to the same argument ν_o. Then all coefficients $r_{i,k}(\nu)$ of the determinant will be functions of only one parameter ν_o:

$$r_{i,k}(\nu)=\Phi_{i,k}(\nu_o). \quad (9)$$

Equation (8) will take the following form:

$$D(\nu_o)=\left|\Phi_{i,k}(\nu_o)\right|=0. \quad (10)$$

By determining the parameters ν_o, we find the values of the critical forces $F_{i,kp}$. The smallest positive value $\nu_{i,kp}$ determines the critical value of any of the vertical forces of the system

$$F_{i,kp}=\frac{\nu_{i,kp}^2 EI_{Ci}}{H_i^2}. \quad (11)$$

5 NEGOTIATING THE RESULTS OF THE STUDY

The compact form of recording the generalized stiffness matrix coefficients [13], made it possible to carry out the deformation and stability calculations in flat multi-story frames of periodic structure using a unified method of forming the equations of the displacement method (1).

A numerical implementation of the proposed method has been performed. For this purpose, an algorithm has been developed and a program in Excel tables has been written. Solving of the system of solving transcendental equations (1) of the deformation calculation is performed by the method of successive approximations. The stability equation (8) is solved by choosing the parameter ν_o. The program makes it easy to analyze the influence of force, geometric and physical-mechanical parameters of model elements on the values of longitudinal forces in the supports of the previously mentioned frames.

To verify the calculation results, a number of test frame calculations for deformation calculation and stability with exact or approximate solutions have been performed [1, 7].

Stability analysis with a geometrical nonlinearity of a flat free frame of a periodic structure (Fig. 4), the model of which was considered in [13].

A physical model of the system “steel structure - fire-retardant coating” has been developed (Fig. 6).

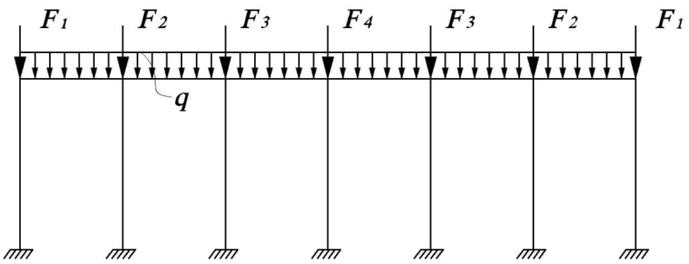

Fig. 4. Model of the frame

Initial data of the model: all columns (height of 6 *m*) and all ledgers (span of 12 *m*) are made of steel with modulus of elasticity $E = 2 \cdot 10^7 \ kN/m^2$. The columns have a rectangular cross-section (a=0.2 *m*; b=0.5 *m*), the moment of inertia of the cross-section of the transom is 2.5 times greater than the moment of inertia of the column cross-section.

At the first stage of the calculations, a system of concentrated vertical forces at the nodes acts on the frame + and uniformly distributed load with intensity $q = 72 \ kN/m$ in all spans.

As a result of the deformation calculation, the internal forces in the elements of the model are determined. The maximum compression force occurs in the second and sixth columns. The values of longitudinal forces in the columns are presented in Table 1 (taking into account the symmetry of the model).

Table 1

Values of longitudinal forces in the frame columns

№		N_1	N_2	N_3	N_4
1	*kN*	680,08	1525,72	1452,04	1468,33
2	proportionality parameter f_i	1,00	2,44	2,24	2,28

In the stability analysis, only the vertical forces (6) are applied to the frame. Their magnitude is proportional to the values of longitudinal forces in the columns, obtained in the deformation calculation (line 2 of Table 1). As a result of the performed calculation, critical values of longitudinal forces in the columns, corresponding to the first two forms of stability loss, were obtained (Fig. 6). The results of the calculations are summarized in Table 2.

Table 2

Critical value of the longitudinal force (in kN)

№		as per the proposed methodology	ANSYS
1	first form of instability	4955,85	4952,74
2	second form of instability	14921,6	14927,1

In order to verify the obtained results, the deformation and stability calculations of this frame model were performed in the ANSYS software package.

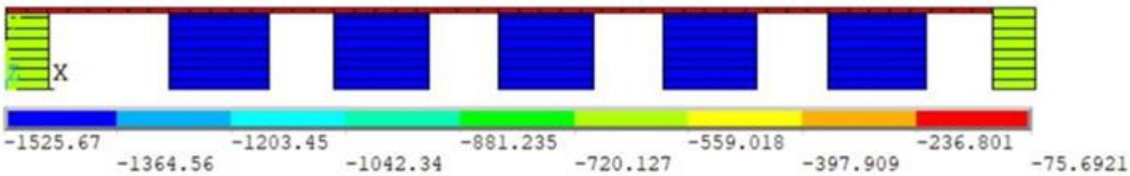

Fig. 5. Longitudinal force diagram (in *kN*)

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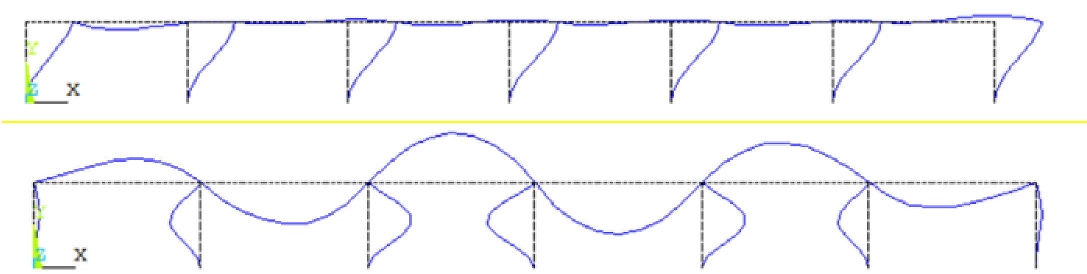

Fig. 6. The first and second forms of frame stability loss (ANSYS)

Comparison of the results of calculations by the proposed method and in the ANSYS software package shows their almost complete coincidence: the difference in the values of longitudinal forces in the frame struts with allowance for geometric nonlinearity is less than 0.01 %; the results of calculations of the first two critical forces differ by 0.06 %.

6 CONCLUSIONS

A methodology for determining the critical values of longitudinal forces in the stability problems of flat multi-span frames by displacement method with regard for geometric nonlinearity is proposed. It allows to use a unified approach to the formation of the systems of the solving nonlinear equations both in the deformation calculation and in the stability calculation. At the first stage, calculation of a frame under the action of a system of concentrated and distributed forces is performed, the ultimate goal of which is to obtain values of longitudinal forces in the frame struts with allowance for geometric nonlinearity. At the second stage of calculations, the critical value of longitudinal force in the frame struts is determined. The algorithm of the proposed method is implemented in the program, which is written in Excel spreadsheets. The verification of the obtained results is confirmed by test calculations for both stages of the calculation. Stability calculations for a flat free one-story frame of periodic structure have been performed using the proposed methodology. Calculations of this frame by the deformed scheme with the subsequent determination of the critical longitudinal force have been performed in the ANSYS software package. The calculation results are compared.

For planar frames of a periodic structure, the suggested method makes it possible to solve the set problems with the same accuracy more economically efficient, since the calculations do not involve the universal computational complexes (ANSYS, LIRA and others).

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