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WAVELET DETECTION OF CRACKS IN FIBER-REINFORCED COMPOSITE BUILDING STRUCTURES

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Abstract. *The aim of this work was to numerically analyze the use of Gabor and Moret wavelets for spatial crack detection in building beams made of composite material with reinforced fibers of different directions. It is indicated that the continuous wavelet transform method is the most effective for detecting damage in building structures. The propagation of shock waves in cantilever beams was modeled using impact mechanical loads. It is found that both Gabor and Morlet wavelets can be used to detect and localize cracks in composite beams of building structures. However, the analysis showed that the Gabor wavelet has a slightly higher accuracy in crack localization. This is due to the fact that the Gabor wavelet has a higher temporal resolution than the Morlet wavelet. The ratio of the response time to the maximum dynamic stress of the crack to the reflection time from the free end of the beam becomes larger with increasing crack depth. This result provides a calibration for choosing the best mother wavelet in the continuous wavelet transform application.*

Key words: *cracks, building structures, wavelet, wave propagation, composite.*

Introduction.

By definition, material destruction is a macroscopic disruption of the material's continuity as a result of certain effects on it [1, 2]. It is assumed that as a result of destruction, the material is divided into two or more parts, which are separated from each other by opposite crack banks. The conditions for crack growth in materials of various natures under the influence of internal and external factors are studied by fracture mechanics.

The key factor determining the destruction of materials is a crack. In turn, a crack is interpreted as a discontinuity of materials, in which one side (length) is much larger than the other (width). A distinctive feature of cracks, according to experts, is their ability to concentrate deformations and stresses in the zones of closure of the banks, which significantly exceed the average values in the volume of the material. The crack was initially assigned the role of an undesirable element, the very existence of which should lead to destruction.

At the same time, it was discovered that as a result of a complex of physical-chemical and physical-mechanical phenomena and processes that occur during the formation of complex materials, internal interfaces are self-generated at each hierarchical level, which are capable of transforming into technological (initial and residual) cracks [3, 4].

All building materials manufactured using composite cement binders are characterized by shrinkage deformations during their hardening and service, the total value of which consists of three types of shrinkage: contraction, moisture and carbonation. Analysis of changes in volume and crack formation in building materials is important due to the fact that the consequence of these changes is a reduction in the operational resources of building compositions, structures and buildings based on them.

Of the available building materials, reinforced concrete is the most durable. A feature of reinforced concrete, as a composite material, is the nonlinear deformation of concrete and reinforcement under load, which contributes to the appearance of shrinkage, temperature, force and corrosion cracks in non-aggressive and aggressive conditions. The presence of cracks has a great influence on the corrosion-mechanical behavior of reinforced concrete structures. At the same time, building codes do not provide a direct assessment of the design residual life of reinforced concrete structures with force cracks, operating in various aggressive environments.

For reinforced concrete structures consisting of multi-component materials, the most reliable scientific results are obtained from full-scale experimental tests, compared to theoretical studies. However, the use of full-scale experiments is not always possible and is quite expensive. This circumstance contributes to the fact that when designing reinforced concrete structures, regulatory documents developed on the basis of experimental studies of direct models of structures are used. At the same time, the expediency of using the dimensional analysis method to study the physical process of crack resistance of reinforced concrete under static force effects has been proven. At the same time, during operation, building structures are subject to repeated force effects. The effect of repeated loads, even in a non-aggressive environment, causes a change in the deformation properties of both concrete and reinforcement.

Despite the great influence of cracks on the durability of reinforced concrete structures, the number of experimental and theoretical studies on the study of force and corrosion cracks on the change in the strength and deformation properties of reinforced concrete structures is very limited.

Among the various testing methods, the wave propagation-based approach has become a reliable and cost-effective non-destructive testing method due to its ease of implementation [5, 6] and its potential use for real-time crack detection in composite structures of building materials. Continuous wavelet transform plays an important role in wave propagation signal processing due to its local and self-adaptive time-frequency properties. The key point in the study of crack formation and dynamics in building materials is the selection of the right mother wavelet. Morlet wavelet is one of the most commonly used wavelets [7] because it has a narrow frequency band and the crack occurrence feature can be better indexed. Gabor wavelet is also popular because of its good time-frequency property.

For composite structures, impact damage is the main cause of failure [8], and cracks are one of the most common damages that shorten the service life of building structures [9]. The objective of this study is to extend the application of CWT to wave propagation in composite beams to detect crack location and damage severity. This study considers pre-cracked cantilever beams under lateral impact. The time-frequency characteristics of Gabor and Morlet wavelets are studied, and their capabilities for crack detection using numerical methods are investigated.

Cracks detection

Continuous wavelet transform is often used to detect signal singularity and check physical damage compared to discrete wavelet transform which is mainly used for data compression and noise removal. The definition of continuous wavelet transform of any square-integrable signal $f(t)$ is described as follows

$$Wf(a,b) = a^{-0.5}\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt, \tag{1}$$

$$Wf(a,b) = (a^{+0.5}/2\pi)\cdot\int_{-\infty}^{+\infty} f^*(\omega)\hat{\psi}(a\omega)\exp(i\omega b)d\omega, \tag{2}$$

where

i is the imaginary unit,

ω the circular frequency,

a and b are the scale and translation parameters.

The Gabor function and its Fourier transform are expressed as

$$\psi_G(t) = \pi^{-0.25}(\omega_0/\gamma)^{0.5}\exp\left[-0.5(\omega_0/\gamma)^2 t^2 \exp(i\omega_0 t)\right], \tag{3}$$

$$\bar{\psi}_G(t) = 2^{0.5}\pi^{+0.25}(\gamma/\omega_0)^{0.5}\exp\left[-0.5(\gamma/\omega_0)^2(\omega-\omega_0)^2\right]. \tag{4}$$

The Morlet function and its Fourier transform are given in the following equations

$$\psi_M(t) = \pi^{-0.25}\exp\left(-0.5t^2\right)\exp(i\omega_0 t), \tag{5}$$

$$\bar{\psi}_M(\omega) = 2^{0.5}\pi^{+0.25}\exp\left[-0.5(\omega-\omega_0)^2\right]. \tag{6}$$

A consequence of the equations is that the time windows of both functions are centered at $t_0 = 0$, while the frequency windows are centered at $\omega = \omega_0$.

For the variances of the Gabor and Morlet functions in the time and frequency domains, the following relationships can be written

$$\sigma_{Gt}^2 = \frac{\int_{-\infty}^{+\infty}(t-t_0)^2|\psi_G(t)|^2 dt}{\int_{-\infty}^{+\infty}|\psi_G(t)|^2 dt} = \frac{\gamma^2}{2\omega_0^2} = 0.36, \tag{7}$$

$$\sigma_{G\omega}^2 = \frac{\int_{-\infty}^{+\infty}(\omega-\omega_0)^2|\hat{\psi}_G(\omega)|^2 d\omega}{\int_{-\infty}^{+\infty}|\hat{\psi}_G(\omega)|^2 d\omega} = \frac{\omega_0^2}{2\gamma^2} = 0.69, \tag{8}$$

$$\sigma_{Mt}^2 = \frac{\int_{-\infty}^{+\infty}(t-t_0)^2|\psi_M(t)|^2 dt}{\int_{-\infty}^{+\infty}|\psi_M(t)|^2 dt} = \frac{\gamma^2}{2\omega_0^2} = 0.5, \tag{9}$$

$$\sigma_{M\omega}^2 = \frac{\int_{-\infty}^{+\infty}(\omega-\omega_0)^2|\hat{\psi}_M(\omega)|^2 d\omega}{\int_{-\infty}^{+\infty}|\hat{\psi}_M(\omega)|^2 d\omega} = \sigma_{Mt}^2 = 0.5. \tag{10}$$

These equations show that both the Gabor and Morlet functions exhibit fairly good time-frequency characteristics, but there is a slight difference between them. Compared to the Morlet function, the Gabor function has a smaller variance in the time domain, but a larger variance in the frequency domain. This means that the temporal resolution of the Gabor function is higher, while the frequency resolution is lower than that of the Morlet function. Thus, the Gabor function can be expected to provide a more accurate time location. This difference in time and frequency resolution results in different performance of the two wavelets in crack detection.

The dispersion wave of deformation detection along the x direction can be written in the form of a Fourier integral as

$$u(x,t)=(2\pi)^{0.5}\int_{-\infty}^{+\infty}A(\omega)\exp[i(\omega t-kx)]d\omega, \quad (11)$$

where

k is the wave number,

ω is the circular frequency.

Thus the continuous wavelet transform of *u* (*x*, *t*) using the Gabor wavelet function can be described as follows

$$Wu(a,b)=a^{0.5}(2\pi)^{-1}\int_{-\infty}^{+\infty}A(\omega)\exp(-ikx+i\omega b)\psi_G(a\omega)d\omega, \quad (12)$$

where $\psi_G(a\omega)$ is localized at $\omega=\omega_0/a$.

Accordingly, the absolute value of the Gabor and Morlet wavelet coefficients are equal to

$$|Wu_G(a,b)|=a^{0.5}\pi^{-0.25}(\omega_0/\gamma)^{0.5}|A(\omega_0/a)|\exp\left[-\frac{(\omega_0/\gamma)^2}{2a^2}\left(b-\frac{x}{c_g}\right)^2\right], \quad (13)$$

$$|Wu_M(a,b)|=a^{0.5}\pi^{-0.25}|A(\omega_0/a)|\exp\left[-\frac{1}{2a^2}\left(b-\frac{x}{c_g}\right)^2\right]. \quad (14)$$

From equations (13) and (14) it is seen that both equations will reach their maximum at time $b=x/c_g$ for a fixed value of *a*. Accordingly, this maximum value is proportional to the amplitude of the wave $A(\omega_0/a)$ at frequency $\omega=\omega_0/a$. For any chosen parameter a, the maximum amplitude of the wavelet coefficient corresponds to the peak at the moment of arrival of the wave component with frequency *f*.

The numerical model considered transverse isotropic cantilever beams with dimensions: 800 mm × 30 mm × 4 mm, consisting of unidirectional glass fiber reinforced epoxy resin. Three types of beams with different fiber directions are considered (shown in Figure 1), in which 00 (sample S1), 450 (sample S2), and 900 (sample S3), indicate the angles between the fiber direction and the axial direction.

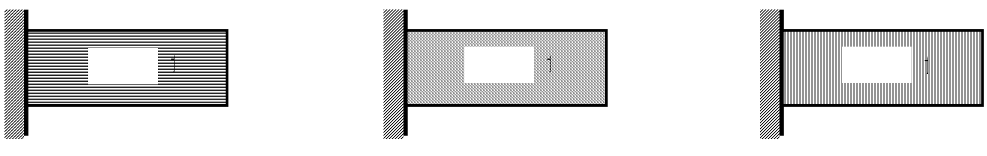

Figure 1 - Types of fiber direction.

Material properties of the beams: density $\rho=1200$ kg·m^{-3}, elastic modulus $E_1=8.2$ GPa, $E_2=7.0$ GPa, shear modulus $G_{12}=1.45$ GPa, $G_{23}=2.60$ GPa and Poisson's ratio $\nu_{12}=0.21$.

For each beam, there is a vertical pre-crack on its upper side, and the pre-crack has a width of 1 mm and a depth of d mm. Five cracked specimens with different crack depths (d = 3, 6, 9, 12, 15 mm: Si1, Si2, Si3, Si4, Si5, i = 1, 2, 3). When the beam is loaded with an impact signal $F(t)$ at its free end, a flexural wave will be excited and propagated along the beam. Once the incident flexural wave encounters the crack, it will be reflected and transmitted. To investigate the flexural wave in the beams, the deformations will be analyzed.

Table 1 - Time for maximum values of dynamic stress

	$t_{1,\max}$, *ms*	$t_{2,\max}$, *ms*	$t_{3,\max}$, *ms*	$t_{4,\max}$, *ms*	$t_{5,\max}$, *ms*
σ_{11}	205	398	621	804	971
σ_{13}	211	380	614	808	965
σ_{21}	217	401	618	816	976
σ_{23}	206	395	620	801	972
σ_{31}	208	394	617	805	979
σ_{33}	212	402	611	809	974

Table 1 shows the response time $t_{k,\max}$ (k = 1, 2, 3, 4, 5) for maximum values of dynamic stress in typical cases: σ_{i1} and σ_{i3} (i = 1, 2, 3). Analysis of the results shows that bending waves in composite beams are dispersive and quite complex.

The bending waves in the three beams are dispersive, so the key issue here is to extract the corresponding wave component for crack detection. Therefore, first, the multi-frequency analysis of the continuous wavelet transform of the deformation signals from 5 to 25 kHz is carried out. Then, the method of crack position detection according to the extracted wave component data is implemented.

Table 2 - Spatial locations of cracks

Specimen	Crack depth ratio	Crack location, *mm*	
		Gabor	Morlet
S11	0.09	143.08	144.16
S12	0.17	152.31	150.87
S13	0.27	147.5	146.72
S14	0.36	144.22	145.64
S15	0.45	148.03	147.33
S21	0.08	144.38	146.06
S22	0.17	151.52	150.88
S23	0.24	144.87	147.43
S24	0.36	132.47	135.09
S25	0.45	136.71	135.18
S31	0.08	146.24	147.05
S32	0.18	142.58	141.16
S33	0.24	148.19	149.25
S34	0.37	146.35	147.23
S35	0.44	152.28	151.16

The deformation signals of the three beams are also processed based on the Morlet wavelet in the same way as based on the Gabor wavelet. It is easy to obtain the arrival time and amplitudes of the incident, crack-reflected, and transmitted waves for the three rays.

Knowing the distance between fixed points and the transit time between the incident and transmitted waves, the group velocity of the wave c_g can be obtained. Then the crack location, defined as the distance between the crack and the nearest point along the wave path, can be calculated as $l = c_g \cdot \Delta t/2$.

The spatial locations of cracks in the building beams of the specified samples, calculated using the Gabor and Morlet wavelet model, are presented in Table 2.

It can be seen from the table that although the dispersive wave in the heterogeneous composite beam is very complex, the crack location can be determined with an acceptable error for engineering purposes. This confirms the good performance of both the Gabor and Morlet wavelets. Moreover, the crack location errors based on the Gabor wavelet are generally smaller than those based on the Morlet wavelet. This is reasonable because the Gabor wavelet function has a slightly higher temporal resolution than the others.

Summary and conclusions.

This methodology presents a crack detection method for composite beams based on continuous wavelet transform. Wave propagation in cantilever beams is investigated using impact experiments. Gabor and Morlet wavelets are applied to complex wave signals to detect crack. The calculation results allow us to state the following conclusions. Both Gabor wavelet and Morlet wavelet can be used to detect and localize cracks in composite beams of building structures. In addition, the results show that Gabor wavelet has a slightly higher accuracy in crack localization. This is due to the fact that Gabor wavelet has a higher temporal resolution than Morlet wavelet.

The results based on the considered wavelets show that the crack-to-reflection ratio becomes larger with increasing crack depth. These results provide a method for estimating crack depth from the crack-to-reflection ratio.

Both Gabor and Morlet wavelets have fairly good time-frequency characteristics. However, the Gabor wavelet has a higher time resolution, while the Morlet wavelet has a higher frequency resolution. This result provides a calibration for choosing the best mother wavelet in the continuous wavelet transform application.

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