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FLEXURAL WAVE PROPAGATION IN ANISOTROPIC FIBER COMPOSITE PLATES

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Abstract. *Monitoring the health of structures consisting of fiber composites can be done in two ways, namely passively, by recording acoustic waves generated by cracks, impact damage, delamination and mechanical shear, and actively, by propagating diagnostic mechanical stress waves and interpreting parameters characterizing the dynamics of the wave propagation. This paper presents a methodology for numerical modeling of flexural wave propagation in a composite plate. In addition, based on the calculation results, recommendations are provided for the use of sensors for detecting damage in local volumes of fiber composite plates based on stress wave parameters. To improve the understanding of the real physical process of flexural wave propagation, a simple model is developed for simulating wave propagation in a plate with different types of boundary conditions. The simulation of waves propagating in the composite was performed using externally applied forces and moments. The results of numerical experiments were used to represent the passive damage growth, which was detected by active acoustic wave generation using piezo-ceramic actuators. Passive detection of acoustic waves was performed using a step function that modeled the acoustic emission from a propagating damage in a local volume of the composite material. A new method for acoustic emission source determination is also presented using the time dynamics of the dominant low-frequency components of the flexural wave mode detected by continuously distributed sensors.*

Key words: *composites, flexural wave propagation, acoustic emission, damage registration, mechanical shear.*

Introduction.

A frequently used technique for damage detection in composite structures is the analysis of the Lamb wave propagation dynamics in the cross-sectional plane [1, 2]. An extension of this technique is the measurement of acoustic emission for monitoring local mechanical damage. Modeling of acoustic and flexural wave propagation is often performed for infinitely sized plates, since obtaining characteristic solutions for the problem of wave propagation in a locally confined medium is a complex mathematical problem. A promising approach in this case is the calculation of normal modes of a closed solution for the propagation of flexural waves in a fiber composite plate [3, 4]. Finite element methods are also used to model wave propagation. However, for this case, the computation time is usually long, and the normal representation of higher frequencies and modes is limited by the large number of elements used.

Impact damage, such as out-of-plane strain source or delamination of the composite material, generates large flexural modes. The results of numerous experiments indicate that the source of flexural modes in the volume of composite samples are only those cracks that are located outside the vicinity of the middle plane. Therefore, the model presented in this study mainly deals with flexural mode acoustic emission signals from impact damage, delaminations and cracks that are not located in the mid-plane [5]. The glass-epoxy composite plate is modeled as quasi-isotropic, and piezoelectric actuators and sensors are modeled on the plate to generate and receive waves.

The aim of this model is to develop a mathematical tool for modeling wave propagation and to facilitate the design of sensors for measuring flexural waves. The sensor elements that are often used in experimental studies to structure a continuous sensor are connected in an $N \times N$ array, which results in the individual signals from the sensors being combined into $2N$ - 1 output arrays. For large array sizes, this approach significantly reduces the number of data acquisition instrument channels needed to monitor the health of a fiber composite structure. The optimal solution in these two approaches is that the continuous sensor is the simplest with a single data acquisition channel, while the array uses more channels to more accurately detect damage. Additional results of the study include a method for localizing the acoustic emission source. The localization technique is based on identifying the dominant low-frequency components of the bending wave. The set of characteristic components was identified based on the time characteristics of the stress measured by four different continuous sensors on four sides of a glass-epoxy plate with a simple support.

Equations of motion and boundary conditions.

The modeling of acoustic emission is based on the equation of motion for a simply supported quasi-isotropic glass-epoxy composite plate. The shape of the pulse of the force excitation of the load is stepwise. The step shape of the mechanical load allows simulating acoustic emission from a propagating damage. The final differential equation is

$$D\nabla^4 w(x, y, t) = -\rho h \ddot{w} + \frac{F(x, y, t)}{ab}, \tag{1}$$

where

$F(x, y, t) = F_1 U(t)\delta(x - x_1)\delta(y - y_1)$ is the step function force;

W is the deflection;

ρ is the mass density;

h is the plate thickness;

a and b are the composite plate length and width.

The plate flexural rigidity is given as

$$D = \frac{Eh^3}{12(1-\nu^2)}. \tag{2}$$

After applying the Navier solution and using the orthogonalization property to separate the spatial and temporal variables, the time equation is obtained. After adding the modal damping, where each damping ratio is specified independently, the time equation can be written as

$$\ddot{a}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{a}_{mn} + \omega_{mn}^2 a_{mn} = F_{mn}, \tag{3}$$

where

$$a_{mn} = f(F_{mn}, \omega_{mn}, \zeta_{mn}, \theta);\ \theta = \tan^{-1}[\zeta(1-\zeta)^{0.5}].$$

The spatial dependence of mechanical stresses in the volume of a composite plate can be specified using the equations

$$\varepsilon_x(x,y,t) = -z\frac{\partial^2 w}{\partial x^2} = -z\sum_n\sum_m a_{mn}(t)\sin\frac{\pi mx}{a}\sin\frac{\pi my}{b}\cdot\left(\frac{\pi m}{a}\right)^2 \tag{4}$$

$$\varepsilon_y(x,y,t) = -z\frac{\partial^2 w}{\partial y^2} = -z\sum_n\sum_m a_{mn}(t)\sin\frac{\pi mx}{a}\sin\frac{\pi my}{b}\cdot\left(\frac{\pi n}{b}\right)^2. \tag{5}$$

As a simple approximation for modeling a unidirectional active fiber composite layer bonded to a plate, the mechanical stress can be approximated using the average strain across the sensor network nodes

$$V_0 = \frac{\varepsilon_1 he}{\varepsilon_s}, \tag{6}$$

where

ε_1 is the average strain;

ε_s is the permittivity.

An analytical model was established for the active propagation of acoustic waves in the plate due to surface-bonded layers of fiber composite. The calculation method allowed us to obtain a closed solution for the propagation of waves in a local volume of the composite material. From the classical theory of vibration of plates, the equation of motion was derived in terms of the internal bending moments of the plate M_x, M_{xy} and M_y, and the moments due to the actuator m_x and m_y, and then expressed in displacement form as

$$D\nabla^4 w(x,y,t) + \rho h\ddot{w} = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2}. \tag{7}$$

The moments associated with the nodes of the sensor network can be expressed as Heaviside step functions

$$m_x = m_x^1[H(x-x_1) - H(x-x_2)]\cdot[H(y-y_1) - H(y-y_2)]f(t) \tag{8}$$

$$m_y = m_y^1[H(x-x_1) - H(x-x_2)]\cdot[H(y-y_1) - H(y-y_2)]f(t), \tag{9}$$

where

$f(t)$ is an explicit function of time, which allows in this case to represent the impulse moment using a delta function.

The next step of the calculation procedure was the analysis of discontinuous functions, the solutions of which described the set of mechanical displacements in the volume of the composite due to step excitation. For the momentum moment, the plate displacement is expressed in terms of the Fourier series (Navier solution) and mode summation, which gives the following

$$w(x,y,t) = \sum_n\sum_m \frac{F_2}{\omega_{mn}}\exp(-\zeta_{mn}\omega_{mn}t)\sin(\omega_{mn}t)\sin\left(\frac{\pi mx}{a}\right)\sin\left(\frac{\pi ny}{b}\right), \tag{10}$$

where

$F_2 = f(\rho, h, a, b)$.

The composite sample in the calculation model has a rectangular shape and is isotropic. The stress distribution inside the plate is assumed to be symmetrical about the neutral axis, and the plate bending creates a linear normal stress distribution. This relationship assumes that the neutral axis coincides with the mid-plane of the plate. In addition, the mass and bending stiffness of the composite volume element are significantly less than those of the entire plate. The mechanical stress in the volume can be integrated to obtain the equivalent bending moment.

The flexural wave number of a fiber composite plate and the corresponding bending wave velocity for a rectangular isotropic plate can be obtained from the equation of motion in polar coordinates and the subsequent Hankel transformation. The bending wave velocity field is dispersive. The wave propagation velocity with higher frequencies is greater than the velocity for lower frequencies. This velocity is defined as

$$c_f = \left(\frac{D}{\rho h}\right)^{0.25} (\omega)^{0.5}, \tag{11}$$

where

$D = Eh^3/[12(1 - \nu^2)]$ is the flexural stiffness of composite plate.

A continuous Fourier transform is performed on the stress-time history obtained as a sum of responses from continuous sensors that are surface-attached to the fiber composite sample. The dispersion dependence of the power spectral density is plotted against frequency to determine the dominant flexural bandwidth. The results are used to determine the peak frequency, especially for the low-frequency components of the flexural mode. Wavelet analysis (scalegram) is then performed to determine and significantly narrow the time bandwidth where the peak amplitude flexural wave hits one part of the continuous sensors.

The continuous wavelet transform $x(t)$ is a time-domain signal processing method that is defined as

$$W_\psi(a,b) = a^{-0.5} \int_{-\infty}^{+\infty} x(t)\Psi^*\left(\frac{t-b}{a}\right) dt. \tag{12}$$

In equation (12), b is the translation indicating the local characteristics of the volume, a is the expansion or scale parameter, $\psi(t)$ is the analysis wavelet, and $\psi^*(t)$ is the complex conjugate of $\psi(t)$. The scale parameter a determines the frequency localization. The analysis wavelet $\psi(t)$ must satisfy the admissibility condition:

$$C_\Psi = \int_{-\infty}^{+\infty} \frac{|\Psi_1(f)|^2}{|f|} df < \infty, \tag{13}$$

where

$\Psi_1(f)$ is the Fourier transform of $\psi(t)$.

Condition (13) is necessary to obtain the inverse wavelet transform. In this case, the Morlet wavelet is defined as

$$\Psi(t) = \exp(j2\pi f_0 t) \cdot \exp\left(-|t|^2/2\right) \tag{14}$$

$$\Psi(t) = \sqrt{2}\exp\left[-2\pi^2(f - f_0)^2\right]. \tag{15}$$

The use of the Morlet wavelet is a necessary condition for obtaining good selectivity of expansion and translation. In practice, the value $f_0 > 5$ is used, which approximately corresponds to the boundary conditions of the first and second kind. It is clear from the definition that the Fourier transform extracts periodic infinite waves from the analyzed function. In contrast, the wavelet transform analyzes the function only locally in fixed regions that are determined solely by the mother wavelet.

Equations (14) and (15) are non-local in the general case. The value of $W_g(a, b)$ at the point (a_0, b_0) depends on $x(t)$ for any value of t. However, as follows from the conditions imposed on the form of the wavelet transform, the function Ψ (t) decays to zero at 1 and ±1. If we assume fast decay, i.e. the values of Ψ (t) are negligible outside the interval (t_{min}, t_{max}), the transform becomes local. Frequency localization can be determined for any region within the fiber composite volume when the wavelet transform is expressed through the Fourier transform:

$$W_\Psi(a,b) = \sqrt{a}\int_{-\infty}^{+\infty} X(t)\Psi^*_{a,b}(af)\cdot \exp(i2\pi f b)df\ . \tag{16}$$

Equation (16) describes the case for which localization depends on the expansion (scale) parameter a. The local resolution of the wavelet transform in time and frequency is determined by the duration and bandwidth of the analysis functions given by $\Delta t = a\cdot t_g$ and $\Delta f = \Delta f_g /a$, where Δt_g and Δf_g are the duration and bandwidth of the wavelet base function, respectively. Thus, in the frequency domain, the wavelet transform has good resolution at low frequencies, and in the time domain, correspondingly, at high frequencies. This qualitative relationship for frequencies is more suitable for detecting transient signals.

The wavelet transform as a signal decomposition tool cannot be directly compared with any time-frequency representation. However, there is a connection between expansion and frequency. The Morlet wavelet analysis corresponds to a functional dependence between the expansion parameter af and the signal frequency fx. This functional dependence can be defined by the relation

$$a_f = f_0 \frac{f_s}{f_w}\cdot\frac{1}{f_x}, \tag{17}$$

where
f_s is the signal sampling frequency;
f_w is the wavelet sampling frequency.

The active wave generation and sensing simulation is performed using a model of the glass–epoxy composite plate. The verification of the computational model was performed, including the case of active wave propagation in the fiber composite using a surface-bonded actuator located in the center of a rectangular plate. The probing was assumed to be carried out in the presence of continuous and array sensors. The first 100 vibration modes were used for the simulations, and the time step was 1 ms. A cross-array sensor architecture was used for the composite sample. The initial analysis was performed for the case of active wave propagation at times of 10–360 ms. Acoustic wave propagation was observed under the condition of antisymmetric Lamb waves (flexural waves) generated by an actuator located in the center of a glass and epoxy plate.

The methodology used to localize the simulated fault location consisted of several computational steps that simulated the measurement process. The elements of the set of voltage-time measurement procedures for four different continuous sensors located on the four sides of the plate were determined separately. Each continuous sensor was modeled as a system of three sensors connected in series to produce a single output. Power spectral density and frequency plots are plotted to determine the dominant lower frequency components of the flexural wave and the corresponding peak frequency. The low frequency component corresponded to the main energy source in the system.

The corresponding calculations for the wavelet scalegram were performed to narrow the initial time interval when the corresponding peak frequency could reach the continuous sensors. As a result of model experiments, the time when the peak bending wave reached the continuous sensor was estimated. The corresponding flexural wave propagation velocity for the peak frequency is then calculated. In this way, the relative distance x_i, $i = 1, 2, 3$ (in percent of the fiber composite specimen length) is calculated as a function of the orientation of the continuous sensors and the simulated local damage location. The time t_i for the flexural wave to reach these sensors for a fixed flexural wave frequency ω_i is then estimated. The simulations were performed for composite specimens (*C1*, *C2*) whose densities differed by a factor of 1.32 and 1.73, respectively. Table 1 presents the calculation details.

Table 1 – Details of damage actual locations

C1			*C2*		
x_i	ω_i, 10^3 Hz	t_i, 10^{-3} s	x_i	ω_i, 10^3 Hz	t_i, 10^{-3} s
0.152	5.863	1.47	0.186	5.863	2.01
0.344	5.863	1.95	0.411	5.863	2.85
0.521	6.018	2.19	0.584	6.018	3.69
0.642	6.018	2.85	0.723	6.018	4.15
0.815	8.214	3.54	0.905	8.214	6.24

Summary and conclusions.

The developed model allowed to calculate the details of the array of mechanical damages with active wave generation and passive probing. Optimization of calculations allowed to reduce the number of data collection channels required to detect damages represented by acoustic emissions or high deformations. The modeling results can be used for active monitoring of mechanical damages in composite plate-type structures. Analysis of bending waves has great potential for finding the location of damages by acoustic emissions from passive continuous sensors.

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