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KINETICS OF SYMMETRIC AND ANTISYMMETRIC MODES IN LAYERED STRUCTURES

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Abstract. *A large number of composite structure testing methods are based on the Lamb wave method. The aim of this work is to develop a numerical model describing the features of the propagation of wave packets associated with the anisotropic characteristics of the local mole of mechanical shears and stresses in the volume of a laminated composite. The numerical method developed in this work allows rapid measurement of the frequency-wavenumber curves, phase velocity and group velocity for the Lamb wave mode A0 in anisotropic material using Snell's law and the time-of-flight concept. The numerical model was based on the assumption that the wave characteristics of the composite are determined by the physical characteristics of the layers and the oscillation frequency. In addition, the characteristics of the wave packets and the direction of their propagation in the composite also depend on the direction of the applied load and the local calculated shape of the volume of the composite structure. The dependences of wave interactions on the component properties, geometry, propagation direction and frequency for waves propagating in multilayer composites are analyzed in detail. It is shown that the exact dispersion relations of symmetric and antisymmetric wave modes in a laminated composite sample can be formulated from the three-dimensional theory of elasticity. This study demonstrates the possibility of extending the revealed features of the Lamb wave mode kinetics to composite laminates with arbitrary stacking sequences. It is found that the Lamb wave velocity can be used to analyze the properties of composite materials. Dispersion curves for the phase velocity of symmetric modes of Lamb wave packets are obtained.*

Key words: *Lamb wave, laminated composites, symmetric modes, dispersion curve, elastic coefficients, shear moduli.*

Introduction.

The widespread use of composite structures in various industries creates an urgent need for testing and evaluation methods. Such methods could monitor and characterize these complex materials. In addition, as a related goal, it is possible to describe the behavior of such materials during their service life. Numerous experiments and theoretical models have resulted in the development of a wide range of analysis methods, which have been categorized as destructive and non-destructive [1, 2]. However, non-destructive methods are often the most attractive, since they do not cause any damage or irreversible changes to the inspected part.

Some non-destructive testing methods are based on Lamb waves. Lamb waves are resonant acoustic excitations guided by the surfaces of a plate structure and are directed along the plate over large distances [3, 4]. These elastic waves are highly dependent on the geometric and material properties of the propagating medium, and thus, the analysis and characterization of Lamb waves propagating in a medium of interest will also help to analyze and understand the medium itself. Non-destructive testing methods using both Lamb waves and impact waves have been widely studied in various experimental and theoretical studies for the purpose of characterizing and evaluating various materials and inspecting various structures for any defects or damage [5, 6]. Improvement and further development of Lamb wave-based methods can be based on the results of experiments using ultrasonic waves and piezoelectric sensors. In addition, the study of elastic properties, temperature fields and moisture distribution in both laminated and reinforced composite samples can also be significantly improved by using wavelet pre-transforms, including Lamb wave transforms [7, 8]. A widely used experimental setup for using ultrasound to investigate composite plates is a completely non-contact hybrid system that uses air and laser propagation paths. The results of the experiments form the basis for Lamb A0 wave modes. The method allows the frequency-wavenumber, phase velocity, and group velocity curves for the Lamb A0 wave mode to be measured in anisotropic material quickly using Snell's law and the time-of-flight concept. Wave interactions depend on the properties of the components, geometry, direction of propagation, and frequency for waves propagating in multilayer composites. The exact dispersion relations of symmetric and antisymmetric wave modes in a plate can be formulated from three-dimensional elasticity theory. The formulation can then be extended to composite laminates with arbitrary stacking sequences. The Lamb wave velocity can be used to analyze the properties of composite materials. The phase velocity depends only on the wave vector, its modulus and, consequently, the direction of wave propagation in the medium. In isotropic materials, phase velocity depends only on the modulus of the wave vector. One of the successful methods for describing mechanical stresses is the construction of the Fourier transform of the Green matrix, the poles of which determine the wave numbers of a composite consisting of N anisotropic layers, as well as the study of the dispersion characteristics of the layered structure [9 - 11]. he algorithm for recursively calculating the Fourier transform of the Green matrix requires only the procedure of inversion of 6 x 6 matrices for any number of layers in the composite. This approach allowed us to obtain curves and surfaces describing the wave numbers, phase velocities, and group velocities of the wave front of Lamb waves. The set of Lamb waves can be considered as a network of wave packets that propagate in symmetric and antisymmetric composites with respect to the direction of propagation and the oscillation frequency. Wave packets propagate in an elastic medium and excite deformations that contain all three components of the displacement vector.

Materials and results

The basic equations of elasticity theory for each of the three-dimensional layers of a non-uniform anisotropic multilayer (packet of N layers) elastic medium have the form

$$\frac{\partial \sigma_{ij}^{(n)}}{\partial x_i} = \rho^{(n)} \frac{\partial^2 u_j^{(n)}}{\partial t^2}, \qquad (1)$$

where $j = 1, 2, 3$; $n = 1, \ldots, N$.

The composite sample has a volume of $\infty \le x, y \le \infty$, $z_{N+1} \le z \le 0$, where z_{N+1} is the distance from the lower boundary of the N-th layer to the upper surface, $z_1 = 0$, $\rho^{(n)}$ is the density of the n-th layer. The layer number will be designated by a superscript. The relationship between mechanical stresses and deformation can be described by the equations of the linear theory of elasticity

$$\sigma_{ij}^{(n)} = C_{ijkm}^{(n)} \varepsilon_{km}^{(n)}, \qquad (2)$$

where $C_{ijkm}^{(n)}$ is a the stiffness tensor of the n-th layer.

When the coordinate system changes, the tensor coordinates change according to the formula

$$C_{ijkm}^{(n)} = a_{pi} a_{qi} a_{rk} a_{ml} C'^{(n)}_{parl}, \qquad (3)$$

where $C'^{(n)}_{pqrl}$ are the coordinates of the stiffness tensor with respect to one coordinate system;

a_{ij} is a 3 x 3 rotation matrix.

The wave characteristics of the composite according to such a model will be determined by the physical characteristics of the layers and the oscillation frequency. In addition, the characteristics of the wave packets and the direction of their propagation in the composite also depend on the direction of the applied load and the local calculated form for the volume of the composite structure.

The calculation model uses a two-dimensional Fourier transform of the displacement vector of the *n*-th layer $u^{(n)}(x, y, z)$

$$U_j^{(n)}(\alpha_1, \alpha_2, z) = F\left[u_j^{(n)}\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_j^{(n)}(x, y, z)\exp[i(\alpha_1 x + \alpha_2 y)]dxdy\,. \tag{4}$$

Fourier transforms allow us to write the following matrix relations

$$S^{(1)}U^{(1)}\Big|_{z=z_1} = Q, \tag{5}$$

$$\frac{dU^{(n)}}{dz} = A^{(n)}U^{(n)}, \tag{6}$$

$$\left[B^{(n)}U^{(n)} - B^{(n+1)}U^{(n+1)}\right]\Big|_{z=z_{n+1}} = 0, \qquad n = 1,\ldots,N-1 \tag{7}$$

$$S^{(n)}U^{(n)}\Big|_{z=z_{n+1}} = 0, \tag{8}$$

where $U(n)$ is the Fourier transform of the vector of displacement components and their ordinary derivatives with respect to z

$$U^{(n)} = \left\{U_1^{(n)}, U_2^{(n)}, U_3^{(n)}, U'^{(n)}_1, U'^{(n)}_2, U'^{(n)}_3\right\}, \tag{9}$$

$$A^{(n)} = \begin{pmatrix} 0 & 1 \\ A^{(n,20)-1}\widetilde{A}_1 & A^{(n,20)-1}\widetilde{A}_2 \end{pmatrix}, \tag{10}$$

where

$$\widetilde{A}_1 = A^{(n,01)}\alpha_1^2 + A^{(n,02)}\alpha_2^2 + A^{(n,03)}\alpha_1\alpha_2 - A^{(n,04)}, \tag{11}$$

$$\widetilde{A}_2 = i\left[A^{(n,11)}\alpha_1 + A^{(n,12)}\alpha_2\right], \tag{12}$$

where $A^{(n,ik)}$ are the matrices with elements $C_{ik}^{(n)}$;

$B^{(n)}$ are matrices that characterize the interaction of layers.

The matrices $A^{(n)}$, $B^{(n)}$ depend only on the material properties of each layer, the oscillation frequency ω and the Fourier variables α_1 and α_2.

The method presented here is related to a linear problem, therefore it is possible to expand the Fourier transform of the displacement component vector with respect to the components of the applied load $Q = \{Q_1, Q_2, Q_3\}$ as follows

$$U^{(n)}(\alpha_1,\alpha_2,z) = U_1^{(n)}(\alpha_1,\alpha_2,z)Q_1 + U_2^{(n)}(\alpha_1,\alpha_2,z)Q_2 + \\ + U_3^{(n)}(\alpha_1,\alpha_2,z)Q_3 = \sum_{p=1}^{3} U_p^{(n)} Q_p \, . \quad (13)$$

For each layer n in the Fourier domain, the solution to this problem can be represented in matrix form

$$U^{(n)}(\alpha_1,\alpha_2,z) = K^{(n)}(\alpha_1,\alpha_2,z)Q(\alpha_1,\alpha_2), \quad (14)$$

where $K^{(n)}$ is the Green's matrix of the problem.

The displacement vector $u(n)$ gives the solution to the problem as a result of the inverse Fourier transform to the displacements $U(n)$ found in the Fourier domain

$$u^n(x,y,z) = F^{-1}\left[U^{(n)}\right] = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U^{(n)}(\alpha_1,\alpha_2,z)\exp[-i(\alpha_1 x + \alpha_2 y)]d\alpha_1 d\alpha_2 \, , \quad (15)$$

The phase velocity of the mode k can be found via

$$c_p^{(k)}(\omega,\gamma) = \frac{\omega}{\zeta^{(k)}(\omega,\gamma)}, \quad (16)$$

and the partial derivative is defined as a group velocity for a direction γ by

$$\frac{\partial\omega}{\partial\zeta^{(k)}(\gamma,\omega_0)} = \frac{\omega_1 - \omega_0}{\zeta^{(k)}(\omega_1,\gamma) - \zeta^{(k)}(\omega_0,\gamma)}, \quad (17)$$

The calculation method is based on the consideration of dimensionless frequencies $\omega h/cT$ and dimensionless velocities c_p/c_T, with which symmetric and antisymmetric modes propagate. The wave packet velocity vectors were located in two directions: in the xy plane at an angle of $c = p/6$ relative to the x axis, and along a straight line at an angle of $c = p/4$ to the x axis in the xy plane. A significant number of studies on the features of Lamb wave propagation in composite structures analyzed the phase velocity graphs depending on the propagation directions for fixed numerical values of the dimensionless frequency xh/c_T (in particular, for the dimensionless frequency, the following values can be specified: 4 and 1.78). The theoretical analysis of the propagation features of Lamb waves is conveniently performed for the following package of dimensionless quantities: dimensionless frequency xh/c_T, dimensionless wave numbers $\zeta \cdot h$, dimensionless phase velocity c_p/c_T and dimensionless group velocity of the wave front c_g/c_T, where x is the angular frequency in rad/s, h is the total thickness of the composite, f are the dimensional wave numbers, c_p, c_g are the phase and group velocities.

The dispersion curves for the propagation of wave groups are characterized by dimensionless propagation velocities cp/c_T of symmetric and antisymmetric modes for dimensionless frequency xh/c_T.

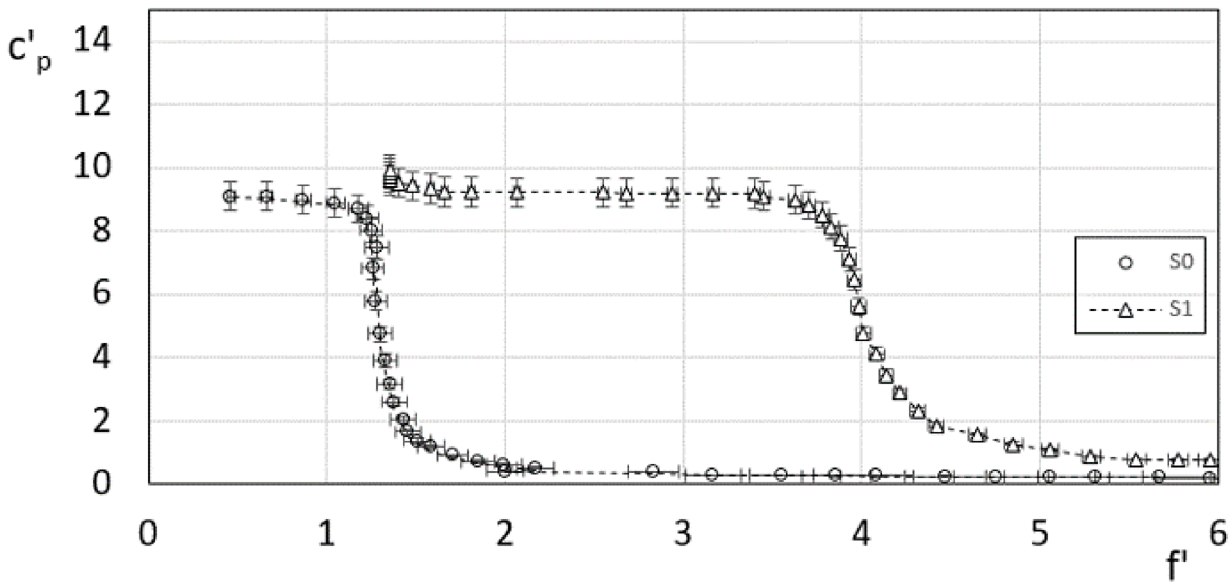

Figure 1 - Phase velocity dispersion curve $c'_p = c'_p(f')$ for *S0* and *S1* modes of Lamb wave

The change in phase velocities depending on the direction of propagation was characterized by the values of the dimensionless frequency xh/cT equal to 3.4 and 1.72, respectively. For the experimental generation of Lamb waves, a laboratory technique of combining a piezoelectric transducer and a wedge was used. The angle of incidence for the generation of Lamb wave modes was determined by Snell's law

$$c_p^{(k)}(\omega,\gamma)=\frac{\omega}{\zeta^{(k)}(\omega,\gamma)}, \tag{18}$$

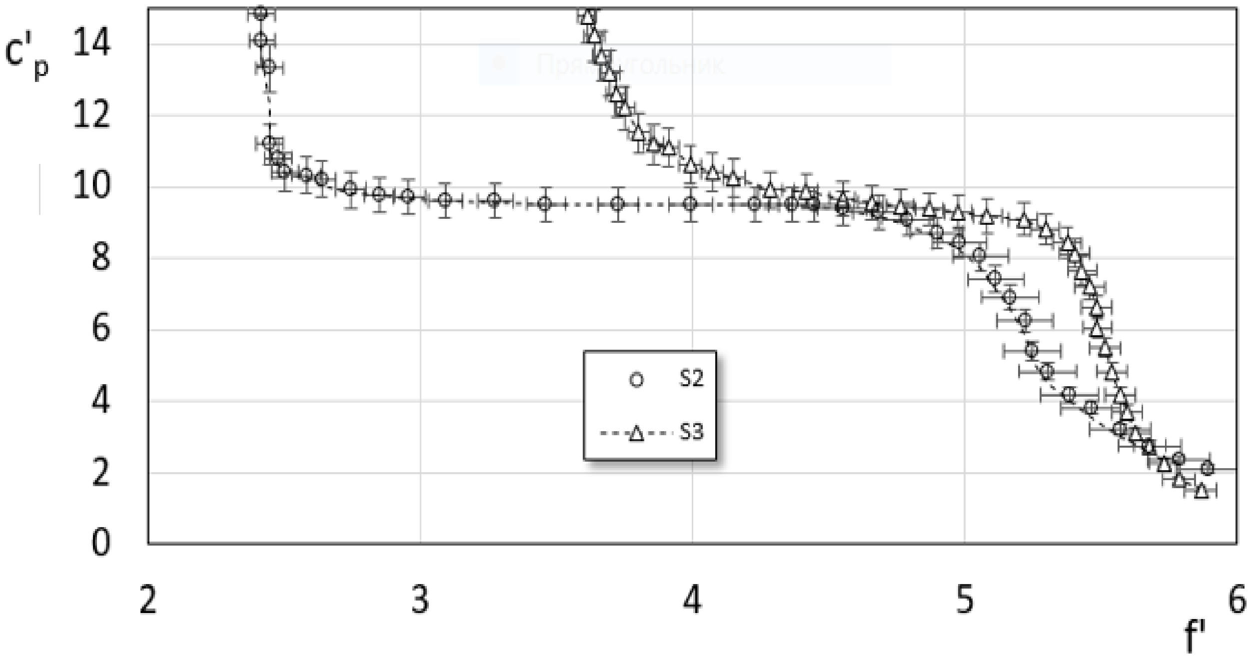

Figure 2 - Phase velocity dispersion curve $c'_p = c'_p(f')$ for *S2* and *S3* modes of Lamb wave

Spectral dependences for the phase velocity of Lamb wave modes S0 – S3 are illustrated in Figures 1 and 2 ($c'_p = c_p / c_{ch}$, $f' = f / f_{ch}$, where c_{ch} and f_{ch} are the characteristic speed and frequency, respectively)

Summary and conclusions.

The A1 mode is an independent incident mode, which is separated from the S1 mode packet by the group delay method. It should be noted, however, that the two-frequency Lamb mode (S2), which is controlled by the fundamental mode S1, depends on the fundamental mode (S1). The group packet of S1 modes, passing through the volume of the composite structure, transfers the fundamental and the two-frequency second harmonic wave. The analysis of the dispersion curves allows us to conclude that the fundamental mode (S1) and the second harmonic mode (S2) have the same phase velocity and group velocity. The detected tendency is similar to the phase condition in resonant vibration, since the generated S2 mode has the same phase and group velocity as the fundamental mode S1. Separation of these two modes in the time domain spectrum is not an ordinary task. However, the study of the frequency spectrum allows us to obtain their amplitudes in the frequency spectrum. The spectra of the fundamental mode S1 and the second harmonic mode S2 are clearly separated in the frequency domain. A large number of experimental and analytical studies on the propagation characteristics of Lamb pulses in the bulk of composites lead to the conclusion that there are two main mechanisms of amplitude attenuation, which are "material attenuation or damping" and "wave packet propagation" corresponding to the wave dispersion effect.

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