UDC 536.24.021

NON-FOURIER HEAT CONDUCTION IN TWO-DIMENSIONAL MEDIA

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Abstract: Real-time heat distribution and phase transformation based on operating conditions and material properties can be estimated using heat equations. The corresponding characteristic functions are used to analyze heat conduction processes in various fields, including laser and electron beam processing. A powerful universal analytical and numerical method that transforms partial differential equations into a coupled system of ordinary differential equations is the wavelet transform method. Fourier and non-Fourier heat equations can be implemented for both equilibrium and non-equilibrium thermodynamic processes, including a wide range of processes such as the two-temperature model, ultrafast laser irradiation, and biological processes. The ultrafast laser heating process of nanofilms is characterized by ultrashort duration and ultrasmall spatial size, in which the classical Fourier law based on the local equilibrium hypothesis is no longer applicable. Based on the Cattaneo-Vernotte model and the double phase delay model, two-dimensional analytical solutions of thermal conductivity in two-dimensional structures under the action of ultrafast laser are obtained using the integral transform method. The results show that there is a thermal wave phenomenon inside the film, which becomes increasingly obvious as the temperature gradient delay time elapses. In this paper, non-Fourier heat conduction problems with temperature and heat flux relaxations are studied based on the wavelet finite element method and solved by the central difference scheme for one-dimensional and two-dimensional media. The heat wave model and the double phase delay model are used to formulate the finite elements, and a new formulation of the wavelet finite element solution is proposed to solve the computational optimization problem. Compared with the current methodologies for the heat wave model and the dual phase delay model, the present model is a direct model that describes the thermal behavior with a single equation with respect to temperature. The developed method can be used for arbitrary shapes. A new iteration update methodology is also proposed for the dual phase delay model to solve the computationally efficient problems. The time iteration algorithms do not use the global stiffness matrix. This allows for optimized calculations. Numerical calculations were performed in comparison with the classical finite element method and the spectral finite element method. The comparisons in accuracy, efficiency, flexibility and applicability confirm that the developed method is an effective and alternative tool for thermal analysis of local volumes of two-dimensional materials.

Keywords: heat wave, heat flux, wavelet-transform, non-Fourier heat conduction, two-dimensional material.

ТЕПЛОПРОВІДНІСТЬ НЕ-ФУР'Є ТИПУ У ДВОВИМІРНИХ СЕРЕДОВИЩАХ

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Анотація: Розподіл тепла в реальному часі та фазове перетворення на основі умов експлуатації та властивостей матеріалу можна оцінити за допомогою рівнянь теплопровідності. Відповідні характеристичні функції використовуються для аналізу процесів теплопровідності в різних областях, включаючи лазерну та електронно-променеву обробку. Потужним універсальним аналітичним і чисельним методом, який перетворює диференціальні рівняння в частинних похідних у пов’язану систему звичайних диференціальних рівнянь, є метод вейвлет-перетворення. Рівняння теплоємності, що описуються та не описуються Фур’є рівнянням можуть бути реалізовані як для рівноважних, так і для нерівноважних термодинамічних

процесів, включаючи широкий спектр процесів, таких як двотемпературна модель, надшвидке лазерне опромінення та біологічні процеси. Надшвидкий процес лазерного нагріву наноплівок характеризується надкороткою тривалістю та надмалим просторовим розміром, у якому класичний закон Фур'є, заснований на гіпотезі локальної рівноваги, більше не застосовний. На основі моделі Каттанео-Вернотта та моделі подвійної фазової затримки методом інтегрального перетворення отримано двовимірні аналітичні рішення теплопровідності у двовимірних структурах під дією надшвидкого лазера. Результати показують, що всередині плівки існує явище теплової хвилі, яке стає все більш очевидним у міру того, як минає час затримки градієнта температури. У цій роботі досліджуються не Фур'є-теплопровідні задачі з релаксацією температури та теплового потоку на основі вейвлет-методу скінченних елементів і розв'язуються центрально-різницевою схемою для одновимірного та двовимірного середовищ. Модель теплової хвилі та модель подвійної фазової затримки використовуються для формулювання скінченних елементів, а також пропонується нове формулювання вейвлетного скінченно-елементного рішення для вирішення проблеми обчислювальної оптимізації. У порівнянні з поточними методологіями для моделі теплової хвилі та моделі двофазної затримки, ця модель є прямою моделлю, яка описує теплову поведінку за допомогою єдиного рівняння щодо температури. Розроблений метод можна використовувати для довільних форм. Нова методологія оновлення ітерації також пропонується для двофазової моделі затримки для ефективного вирішення обчислювальних проблем. Алгоритми часових ітерацій не використовують глобальну матрицю жорсткості. Це дозволяє оптимізувати розрахунки. Чисельні розрахунки проводились у порівнянні з класичним методом скінченних елементів та спектральним методом скінченних елементів. Порівняння точності, ефективності, гнучкості та застосовності підтверджують, що розроблений метод є ефективним та альтернативним інструментом для термічного аналізу локальних об'ємів двовимірних матеріалів.

Ключові слова: хвиля тепла, тепловий потік, вейвлет-перетворення, теплопровідність не Фур'є типу, двовимірний матеріал.

1 INTRODUCTION

The treatment of some film materials with high-intensity and ultrashort laser radiation leads to the appearance of microscale hot spots of heat transfer. In this case, experiments indicate the appearance of sharp wave fronts responsible for the temperature rise, which are difficult to interpret by the classical Fourier model. The macroscopic heat wave model can be considered as the first attempt to describe heat transport at the microscale. A subsequent modification led to the hyperbolic equation of thermal state and assumed the description of heat transport by a wave with a finite velocity. A number of attempts to describe heat transport at the microscale were supplemented by the Cattaneo-Vernotte model and the dual-phase model with delay.

However, the two-stage model and the later proposed pure phonon field model suggest that the microscale thermal behavior follows neither the pattern given by the thermal wave model nor the Fourier diffusion model. To fill the gap between the microscopic and macroscopic theories, a double-phase-lag model was proposed according to two time constants in the thermal evolution equation. The double-phase-lag model aims to eliminate the precedence assumption made in mesoscale heat transport models. The basic procedure in these considerations is to fix the cause-and-effect relationship between the temperature gradient and the heat flux. It should be emphasized that the models for the meso- and microscales are derived using a Taylor series expansion. In fact, this method of derivation is incompatible with the second law of thermodynamics. Moreover, the equations of the double-phase-delay model are a special, linearized version of the Jeffrey equation and use interdependent parameters.

2 ANALYSIS OF LITERATURY DATA AND RESOLVING THE PROBLEM

Compared to the parabolic diffusion equation, the Fourier model and the double phase delay model are hyperbolic methods. As a result, there is a resurgent interest in solving heat equations given by a model that takes into account the finite speed of heat wave propagation in the tested media. The set of numerical schemes for solving the heat conduction equation can be divided into both analytical and numerical solutions. The most widely used methodologies are those using the Laplace transform [1, 2], Green function [3, 4] and the integral equation method [5, 6]. The rather complex mathematical transformations that are necessary in analytical methods represent the main obstacles to their practical application. Due to the complexity of the hyperbolic equation, which leads to ambiguity in the tested domain, only very few simple cases can be solved analytically. Consequently, numerical solutions and methods attract more attention.

However, the exact solution for the two-dimensional film material is still sometimes not easy to obtain. One of the most common techniques for analyzing this problem is the finite difference method [7-10]. A significant improvement of this approach was the finite difference algorithm, based on which a convergent three-level finite difference scheme was developed [11-13] and a high-accuracy finite difference method for solving the two-dimensional problem [14-16]. The discontinuous finite difference model [17, 18] has enabled the analysis of heat wave propagation in one-dimensional and two-dimensional media using a double-delayed phase model. Modification of the finite difference method by wavelet transforms is not widely used in analytical methods of studying heat conduction processes in two-dimensional structures. However, it should be noted that the effectiveness of wavelet transforms has been previously tested for dynamic analysis and elastic wave problems.

This study aims to develop a wavelet transform formulation for the finite difference method and corresponding solution methodologies for non-Fourier heat conduction, in particular, for the heat wave model and the double phase delay model.

3 PURPOSE AND OBJECTIVES OF THE STUDY

Since the heat diffusion equation is parabolic in nature, it is easy to see from the idea of wave motion that this equation implies an infinite speed of heat wave propagation. In turn, this indicates that a local change in the heat flux q spatial r and temporal t coordinatescan lead to an instantaneous perturbation of the temperature field Θ. It was verified that the conclusion is inconsistent with experiments. With the development of materials processing using pulsed sources and the requirement of laser-induced guide waves in structural health monitoring, the classical Fourier law has been shown to be inadequate in modeling high-frequency response. The above problems have led to many attempts to improve the classical model, the most famous of which is probably the thermal wave model, which takes into account thermal "inertia".

In this model, the approximation of the heat transfer process can be described by a Jeffreys-type constitutive relation

$$\tau_0\Theta_{tt}(r,t)+\Theta_t(r,t)-\alpha\Delta\Theta=Q(r,t)/(\rho c)+\tau_0 Q_t(r,t)/(\rho c),$$

where τ_0- is the relaxation time; $Q-$ depicts the heat source; $\rho-$ is the density; $c-$ is the specific heat of the material; $\alpha=\kappa/(\rho c)-$is the thermal diffusivity; $\kappa-$ is the conductivity for thermal medium.

It should be noted that this equation cannot be considered as a real approximation of the single-phase delay relationship, which leads to ill-posed problems. A series of experimental studies have confirmed that the thermal wave model performs better than the classical Fourier law in numerical prediction. At the same time, it has been found that the model only takes into account fast transient effects, but not microstructural interactions. These two effects can be reasonably represented by the double-phase-delay model as the relationship between the heat flux q and the temperature gradient $\nabla\Theta$

$$\tau_0\Theta_{tt}(r,t)+\Theta_t(r,t)-\alpha\Delta\Theta(r,t)-\alpha\tau_\Theta[\Delta\Theta(r,t)]_t=Q(r,t)/(\rho c)+\tau_0 Q_t(r,t)/(\rho c),$$

where τ_Θ is the delay time caused by the micro-structural interactions (phase-lag of the temperature gradient).

The extreme nature of the thermal parameters used in numerical calculations of pulsed thermal action by a laser leads to difficulties at the stage of modeling the processes. As a rule, the models of the thermal wave and double phase delay are usually transformed into the corresponding normalized forms. At the first stage, the parameters of the excited pulse are subjected to a non-dimensionalization procedure. Next, a Gaussian profile is used to model the light intensity of laser pulses

$$Q=\frac{(1-R)I_0}{\sqrt{\pi}t_p}\exp\left(1-t^2/t_p^2\right),$$

where $R-$is the reflectivity of irradiated surface; I_0- is the laser output intensity; t_p- is the full-width-at-half-maximum of pulse.

The set of dimensionless parameters of length (X,Y,R_c), time $(\gamma,\gamma_1,\gamma_2)$, temperature (β), heat flux (φ), and heat source (Ψ) were used in this study

$$X=\frac{x}{2\sqrt{\alpha\tau_0}},\ Y=\frac{y}{2\sqrt{\alpha\tau_0}},\ R_c=\frac{r_c}{2\sqrt{\alpha\tau_0}},\quad \gamma=\frac{t}{2\tau_0},\ \gamma_1=\frac{t_p}{2\tau_0},\ \gamma_2=\frac{\tau_\Theta}{2\tau_0},$$

$$\beta=\sqrt{\frac{\pi\tau_0}{\alpha}}\frac{\kappa\left(\Theta-\Theta_0\right)}{\left(1-R\right)I_0},\quad \phi=\frac{\tau_0\sqrt{\pi}}{\left(1-R\right)I_0}q,\ \psi=\frac{2\tau_0\sqrt{\pi\alpha\tau_0}}{\left(1-R\right)I_0},$$

where T_0- is the reference temperature.

In this case, the dimensionless model with a phase delay of the Jeffreys constitutive type is rewritten as

$$\beta_{\gamma\gamma}+2\beta_\gamma-\Delta\beta-\gamma_2[\Delta\beta]_\gamma=2\psi+\psi_\gamma,$$

and for the value Ψ the following relation can be written

$$\psi=\frac{\sqrt{\alpha\tau_0}}{\gamma_1}\exp\left(1-\gamma^2/\gamma_1^2\right).$$

Both models contain partial differential equations which, after transformation, can be used in the finite difference method to describe heat transfer processes for boundary conditions of different types.

The finite element method involves first partitioning the domain Ω into a grid in terms of a set of non-overlapping subdomains Ω_e. Subsequently, each subdomain is mapped onto a unit interval taking into account the dimensionality of the problem being analyzed. The finite element method is complemented by wavelet transforms. In the unit interval, some wavelets of the m-th order B-spline j on the interval $\eta_{j,m,k}\left(\xi\right)$ are used to construct wavelet finite element formulations for the heat transfer problem. According to the m-th order B-spline functions corresponding wavelets with m-th order $\left(BSWI_{mj}\right)$ can be determined. In this case, the inner B-spline describes a mesh of m segments for the cases $A1$ (no boundaries), $A2$ (internal nodes), and $A3$ (presence of one boundary):

$$A1: x_{-m+1}^j=x_{-m+2}^j=\ldots=x_0^j=0,$$

$$A2: x_k^j=k2_{-j}^k, k=0,1,\ldots,2^j,$$

$$A3: x_{2j+1}^j=x_{2j+2}^j=\ldots=x_{2j+m-1}^j=1.$$

An arbitrarily specified scale j immediately fixes the discretization step, which in this case is equal to $1/2^j$. At the next stage, in order to introduce at least one internal B-spline function into the working area, the following condition is checked for feasibility

$$2^j\geq 2m-1.$$

Assuming that j_0 is the initial scale, for each previously considered case $\left(A_i, i=1,2,3\right)$ for $j\geq j_0$ we get:

$$A1: \eta_{m,k}^j\left(\xi\right)=\eta_{m,k}^{j_0}\left(2^{j-j_0}\xi\right), k=-m+1,\ldots,-1;$$

$$A2: \eta_{m,k}^j\left(\xi\right)=\eta_{m,2j-m-k}^{j_0}\left(1-2^{j-j_0}\xi\right), k=2^j-m+1,\ldots,2^j-1;$$

$$A3: \eta_{m,k}^j\left(\xi\right)=\eta_{m,0}^{j_0}\left(2^{j-j_0}\xi-2^{-j_0}k\right), k=0,\ldots,2^j-m.$$

The scaling functions $\eta_{j,m,k}(\xi)$ satisfy the relation:

$$\eta_{m,k}^{j} = \eta_{m,k}^{j_0}\left(2^{j-j_0}\xi\right), k = -m+1,...,-1;$$

$$\eta_{m,k}^{j} = \eta_{m,2j-m-k}^{j_0}\left(2^{j-j_0}\xi\right), k = 2^{j}-m+1,...,2^{j}-1;$$

$$\eta_{m,k}^{j} = \eta_{m,0}^{j_0}\left(2^{j-j_0}\xi - 2^{-j_0}k\right), k = 0,...,2^{j}-m.$$

The system of primary scaling functions allows for a full scaling of the wavelet transforms, which fix both the main interpolating function and the shape function in the parent wavelet. For the two-dimensional case, we then define the horizontal and vertical interpolating vectors based on the equation:

$$\eta_{\xi} = \left\{\eta_{m,-m+1}^{j}(\xi)\eta_{m,-m+2}^{j}(\xi)...\eta_{m,2^{j}-1}^{j}(\xi)\right\}, \eta_{\chi} = \left\{\eta_{m,-m+1}^{j}(\chi)\eta_{m,-m+2}^{j}(\chi)...\eta_{m,2^{j}-1}^{j}(\chi)\right\},$$

where ξ, η belong to the interval [0, 1], which depict the normalized *x* and *y* coordinates, respectively.

The two-dimensional interpolating function is formulated based on the Kronecker product between two vectors in the characteristic equation. The basic procedures of two-dimensional analysis can be carried out for the basic functions of wavelet transforms. Within the framework of the finite element method, the unknown continuous function of the temperature field $\Theta(\xi,\chi,t)$ can be interpolated in the elementary domain as

$$\Theta(\xi,\chi,t) = S\,\Theta^{e},$$

where S is the interpolating function; Θ^{e} is the nodal temperature in an element.

The analysis of the variants of constructing the computational grid showed that there is more than one node in the element. Only for such a case the interpolating function and nodal temperature can be written in matrix form. In this work, the third-order wavelet transform function is chosen as the interpolating function S.

The physical field of two-dimensional composite structures can be written in terms of wavelet coefficients. In this case, for the convergence of the calculation procedure in wavelet interpolations, an additional transformation matrix is required. This matrix Θ is used to transform the wavelet coefficients into the physical domain. After the specified steps, the interpolation of the value S can be carried out, namely

$$\eta\,\Theta = S.$$

and for the transformation matrices we get

$$\hat{\Theta} = \left\{\eta_{\xi}^{\Theta_1}(\xi_1), \eta_{\xi}^{\Theta_2}(\xi_2)...\right\}^{-\Theta} \otimes \left\{\eta_{\chi}^{\Theta_1}(\chi_1), \eta_{\chi}^{\Theta_2}(\chi_2)...\right\}^{-\Theta}.$$

The system of partial differential equations fixed for the heat wave model is transformed into a wavelet transform formula for finite elements using a trial function. The requirement of two-order continuity of β, namely the $\Delta\beta$ component, complicates the choice of the trial function in the wavelet transform. Therefore, the weak form is usually used. By writing the characteristic equation with the trial function ϑ and integrating it by parts over the region of interest Ω, we get

$$\int_{\Omega}\beta_{\gamma\gamma}d\Theta + 2\int_{\Omega}\beta_{\gamma}d\Theta + \int_{\Omega}\nabla\vartheta\nabla\beta d\Omega = \int_{\Omega}\vartheta\left(2\psi + \psi_{\gamma}\right)d\Omega..$$

The weak form of the heat wave propagation model can be obtained in matrix form based on Hamilton's principle, namely

$$P\beta_{\gamma\gamma} + Y\beta_{\gamma} + U\beta = W,$$

where the matrices included in this equation can be defined as follows

$$P = \sum_{e}\sum_{i}^{n+1}\sum_{j}^{n+1}\omega_i\omega_j S^T S \det(J), \quad Y = \sum_{e}\sum_{i}^{n+1}\sum_{j}^{n+1}2\omega_i\omega_j S^T S \det(J),$$

$$U = \sum_{e}\sum_{i}^{n+1}\sum_{j}^{n+1}\omega_i\omega_j \nabla S^T \nabla S \det(J), \quad P = \sum_{e}\sum_{i}^{n+1}\sum_{j}^{n+1}\omega_i\omega_j S^T \left(2\psi + \psi_{\gamma}\right)\det(J),$$

where e is the symbol that defines the total number of finite elements used in the simulation; i and j are the element indices that correspond to different directions in the two-dimensional medium; ω_i and ω_j are the corresponding weights of the Gaussian integrations; J is the Jacobian matrix.

The methodology for calculating the reduced matrix parameters refers to the basic theory of the finite element method. The methodology for calculating the reduced matrix parameters refers to the basic theory of the finite element method. The structure of the equation containing the matrix parameters is the same as that of a typical wave propagation equation or dynamic reactions in elastic interactions. Based on these assumptions, it can be stated that the temperature change propagates in a wave-like mode.

The same weak form process for the heat wave propagation model is used to construct the corresponding weak form for the double phase delay model, where P, Y, Z, U and W are defined by the following relation

$$P\beta_{\gamma\gamma} + \left(Y + Z\right)\beta_{\gamma} + U\beta = W.$$

The matrix Z, which defines the damping parameters, is essentially the key element that contains the difference between the double phase delay model and the model of body wave propagation in a two-dimensional medium. The relaxation time ratio $\gamma_2 = \tau_{\Theta} / \tau_0 \gamma_2$ determines the properties of Z, matrix in double phase delay model

$$Z = \gamma_2 \sum_{e}\sum_{i}^{n+1}\sum_{j}^{n+1}\omega_i\omega_j \nabla S^T \nabla S \det(J) = \gamma_1 U.$$

The equations for W and Z represent the basic formulation of the solution of wavelet transforms for the heat wave model and the double phase delay model. However, numerical experiments have shown that these basic formulations are only suitable for calculations that rely on a computational grid with small cells. As a result, this places a strong limitation on the number of degrees of freedom.

The mode superposition scheme and the central time difference integration scheme are well suited for solving the systems of equations of the double phase delay model. These schemes, however, are limited in the case of small degrees of freedom. To analyze this problem, a 1000-degree-of-freedom structure could be considered. Weak forms can be used to obtain matrices P, Y, Z and U of size 1000 × 1000 and a vector W of size 1000 × 1. After this, the superposition of modes can be performed. In general, for the inverse matrix of 1000 × 1000, the improved numerical method of Lankoz should be used. It should be noted that for the problem of heat wave propagation, direct time iteration, namely, the scheme of integration over the central difference time, is more effective. Let us write the basic equations for the propagation of a heat wave in accordance with the central difference time integration scheme:

$$P\left(\frac{\beta_{\gamma+\Delta\gamma} - 2\beta_{\gamma} + \beta_{\gamma-\Delta\gamma}}{\Delta\gamma^2}\right) + Y\left(\frac{\beta_{\gamma+\Delta\gamma} - \beta_{\gamma-\Delta\gamma}}{2\Delta\gamma}\right) + U\beta_{\gamma} = W\beta_{\gamma},$$

$$\left(\frac{1}{\Delta\gamma^2}P+\frac{1}{2\Delta\gamma}Y\right)\beta_{\gamma+\Delta\gamma}=W\beta_{\gamma}-U\beta_{\gamma}+\left(\frac{2}{\Delta\gamma^2}P\right)\beta_{\gamma}-\left(\frac{1}{\Delta\gamma^2}P-\frac{1}{2\Delta\gamma}Y\right)\beta_{\gamma-\Delta\gamma}.,$$

where $\Delta\gamma$ is the step between the neighbor integration slice in time domain.

Accordingly, for the dual phase delay method the last equation should be modified using the following substitutions

$$\frac{1}{\Delta\gamma^2}P+\frac{1}{2\Delta\gamma}Y\to\frac{1}{\Delta\gamma^2}P+\frac{1}{2\Delta\gamma}Y+\frac{\gamma_2}{2\Delta\gamma}U,\ \ \frac{1}{\Delta\gamma^2}P-\frac{1}{2\Delta\gamma}Y\to\frac{1}{\Delta\gamma^2}P-\frac{1}{2\Delta\gamma}Y-\frac{\gamma_1}{2\Delta\gamma}U.$$

For the analysis of heat wave propagation, it is important to fix both the boundary and initial conditions. Thermal boundary conditions can be divided into three types: the Dirichlet condition for temperatures of some regions (condition 1); the Neumann condition for heat flows; (condition 2); a mixture of the first and second boundary conditions (condition 3).

It is worth noting that the exact definition of the boundary condition of the second type in the given calculation algorithm is impossible. The reason for this statement is that the heat flux in non-Fourier problems is not directly proportional to the temperature gradient, especially in heterogeneous materials or for low-temperature phenomena. Implicit calculation scheme, allows to describe experimental results beyond Fourier. In this technique, the initial condition constraint is to set the initial value to β_{-1} and β_0, and thus the initial value of temperature and heat flux can be controlled. Of course, a higher-order modification of the first and second derivatives of temperature can be considered in the calculation scheme. But such a calculation technique is not accurate enough. For this reason, the heat flux is usually considered as an independent state variable in the calculations.

Conditional stability is a drawback of the central difference method. This stability can be formulated as a requirement that the time step length $\Delta\gamma$ be less than some critical value γ_{cr} closely related to the dynamic properties of the discretized system

$$\Delta\gamma\le\Delta\gamma_{cr}=2/\omega_n,$$

where ω_n- is the shortest period of eigenvalue of the discrete system.

4 RESEARCH RESULTS

Model calculations for the case of heat wave propagation assumed the use of the boundary condition of the second type. In particular, the heat flux through local boundary surfaces was zero. For the one-dimensional case, the boundary is set at the two ends of the domain. The Neumann boundary in the case of a two-dimensional medium was limited by the lateral surfaces. For both the one-dimensional and two-dimensional cases, an initial condition is used for which the initial changes in temperature and heat flux were negligible.

The calculation of the physical parameters of heat transfer before the non-dimensionalization procedure was performed for the following characteristic values. Thermal and kinematic parameters: $\alpha=0,23\cdot10^{-4}m^2/s,\tau_0=0,172ps.$ Geometrical characteristics: $x=5\,mn$ (equivalent to 1.257 in the dimensionless domain) for the 1D case, radius $r=6\,nm$ (equivalent to 1.508 in the dimensionless domain) for the 2D case. Energy characteristics: excitation parameter, $\tau_p=100\ fs$ (equivalent to $\gamma_1=0,29$), the reflectivity of the irradiated surface is simply assumed to be $R=0$. Spatial and temporal computational grid characteristics: for the one-dimensional case, $t=3\ ps$ (equivalent to $\gamma=8,72$), divided into 10,000 time steps; for the two-dimensional case, $t=1,5\ ps$ (equivalent to $\gamma=4,3604$), divided into 10,000 time steps.

Temperature changes at the excited point ($X_0 = 0$), the middle of the medium ($X_1 = 0,657$) and at the end of the one-dimensional region ($X_2 = 1,293$) were calculated using the thermal wave propagation and double phase delay methods. Tables 1-3 illustrate the calculated dependencies $\eta = \eta(\gamma)$ obtained using wavelet transforms for cases *C*1 – *C*6 ($\gamma_1 = 0; 0,1; 0,4; 0,8; 1,2;$ and 1.5, respectively).

Table 1

Wavelet performances for different γ_1 (point X_0)

γ	η					
	C1	C2	C3	C4	C5	C6
0	0	0	0	0	0	0
0.5	1.104	1.259	1.340	1.365	1.418	1.468
1	0.712	0.867	0.939	0.984	1.021	1.077
1.5	0.562	0.717	0.794	0.830	0.870	0.921
2.0	0.482	0.641	0.710	0.752	0.787	0.838
2.5	0.401	0.554	0.622	0.663	0.717	0.763
3.0	0.367	0.526	0.603	0.628	0.675	0.736
3.5	0.322	0.476	0.542	0.592	0.626	0.675
4.0	0.295	0.459	0.535	0.568	0.604	0.648
4.5	0.241	0.409	0.470	0.503	0.547	0.602
5.0	0.237	0.399	0.473	0.514	0.547	0.590
5.5	0.230	0.394	0.466	0.491	0.546	0.599
6.0	0.228	0.390	0.452	0.506	0.548	0.580
6.5	0.221	0.387	0.451	0.488	0.523	0.574
7.0	0.216	0.373	0.437	0.496	0.528	0.575
7.5	0.214	0.372	0.446	0.484	0.518	0.569
8.0	0.211	0.371	0.445	0.478	0.530	0.569

Table 2

Wavelet performances for different γ_1 (point X_1)

γ	η					
	C1	C2	C3	C4	C5	C6
0	0	0	0	0	0	0
0.5	0	0	0	0	0.026	0.031
1	0	0	0.042	0.050	0.054	0.058
1.5	0.081	0.110	0.253	0.293	0.405	0.471
2.0	0.342	0.264	0.407	0.454	0.554	0.631
2.5	0.275	0.252	0.390	0.441	0.538	0.611
3.0	0.264	0.247	0.390	0.432	0.531	0.605
3.5	0.253	0.245	0.381	0.432	0.543	0.607
4.0	0.258	0.241	0.386	0.429	0.533	0.602
4.5	0.256	0.237	0.377	0.423	0.526	0.598
5.0	0.252	0.235	0.368	0.423	0.527	0.596
5.5	0.250	0.232	0.376	0.418	0.529	0.581
6.0	0.248	0.230	0.375	0.422	0.528	0.572
6.5	0.241	0.228	0.364	0.421	0.511	0.566
7.0	0.239	0.226	0.357	0.417	0.509	0.564
7.5	0.236	0.224	0.361	0.413	0.507	0.560
8.0	0.233	0.221	0.366	0.403	0.501	0.558

Table 3

Wavelet performances for different γ_1 (point X_2)

γ	η					
	C1	C2	C3	C4	C5	C6
0	0	0	0	0	0	0
0.5	0	0	0	0	0	0.020
1	0	0	0	0	0.021	0.044
1.5	0	0	0.022	0.034	0.063	0.085
2.0	0	0	0.036	0.082	0.127	0.234
2.5	0.032	0.083	0.133	0.251	0.337	0.399
3.0	0.226	0.187	0.238	0.338	0.447	0.502
3.5	0.241	0.236	0.301	0.399	0.495	0.540
4.0	0.252	0.261	0.313	0.427	0.515	0.564
4.5	0.260	0.270	0.340	0.438	0.531	0.583
5.0	0.262	0.272	0.334	0.442	0.528	0.586
5.5	0.260	0.273	0.337	0.441	0.537	0.592
6.0	0.258	0.271	0.327	0.426	0.541	0.596
6.5	0.256	0.268	0.323	0.429	0.523	0.598
7.0	0.254	0.265	0.329	0.416	0.518	0.602
7.5	0.253	0.264	0.321	0.433	0.527	0.604
8.0	0.252	0.263	0.326	0.413	0.519	0.606

5 DISCUSSION OF RESEARCH RESULTS

The calculation results indicate a shift of the extreme values of η relative to $X=0$ to the region of large values γ. In addition, the increase in dispersion near the maximum $\eta=\eta(\gamma)$ is smeared with increasing γ_2. The diffusive behavior becomes more dominant. The tendency for the dispersion to increase becomes more and more pronounced for the responses at $X=X_1$ and $X=X_2$. At the midpoint $(X\approx X_{\max}/2)$ the responses for $\gamma_2=0$ exhibit typical wave behavior similar to an elastic wave, and the wave arrival time is instinctively close to $\gamma=1,25$. The propagation pattern of the thermal wave corresponds to the existence of two regimes for the diffusion $(\gamma_2=0,5)$ and superdiffusion cases $(\gamma_2=1,5)$. For these regimes, the time for the wave to completely pass through the sample is calculated with a large error, since the thermal behavior is not wave-like. Comparison of the responses at $X=0,6283$ with $X=1,257$ allows us to state that the temperature disturbance arrives at almost the same time for these two positions. Consequently, the speed of the heat wave is large enough for the diffusion and superdiffusion cases.

The changes in the temperature distribution in the computational domain have a clearly expressed wave-like character. In this case, the pulsed thermal disturbance propagates in the wave form. The value $\gamma=2,16$ corresponds to a clear circular wave front, on which the energy of the pulsed thermal disturbance is mainly concentrated. With increasing dimensionless time ($\gamma=4,38$), the thermal energy is slowly absorbed by the medium.

Due to the increase of γ_2, the matrix Z plays a more important role in the damping term. As a consequence, the temperature changes in the region after the wave front become sharper compared to the wave-like behavior. When $\gamma_2=0,5$, all wave-like features disappear, the disturbance caused by the pulse is completely transferred by diffusion. The wave front shape is smoothed out and the characteristic temperature in this case is always the hot spot.

Increasing γ_2 to 1.5 is the reason for the occurrence of the super-diffusion regime. Compared with normal diffusion, this value provides a higher diffusion rate at an early stage.

6 CONCLUSIONS

In this paper, an improved formulation of finite difference wavelet transforms is developed to perform numerical experiments to describe the thermal characteristics in one-dimensional and two-dimensional media. The simultaneous use of the central difference scheme in time and the wavelet interpolation technique for spatial coordinates allowed us to develop a hyperbolic model of thermal conductivity in two-dimensional structures. In order to verify the calculations, the proposed algorithm was tested by comparing with classical finite difference and spectral finite difference methods. The comparison was made taking into account the parameters of accuracy and efficiency.

For the case of 2D media, different mesh geometries were tested for different types of boundary conditions. The improved methods of describing thermal wave propagation and double phase delay can be considered as an alternative tool for thermal analysis. It should be noted that the use of the heat flux boundary condition in this calculation method requires further development due to the accumulation of calculation errors of the finite difference method. The wavelet-transforms of the state variable expressed in terms of finite differences can provide a solution to this problem.

7 ETHICAL DECLARATIONS

The author have no relevant financial or non-financial interests to report.

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For references:
O. Pysarenko. (2025). Non-Fourier heat conduction in two-dimensional media. Mechanics and Mathematical Methods. VII (1). 90 –102.

Для посилань:
Писаренко О. М. Теплопровідність не-Фур'є типу у двовимірних середовищах. Механіка та математичні методи, 2025. Т. VII. №. 1. С. 90-102..