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PROPAGATION OF LAMB WAVES IN LAMINAR COMPOSITES

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Abstract. *Low cost, the possibility of online monitoring and high sensitivity distinguish the method of structural monitoring using Lamb waves from other available methods. Structural analysis based on Lamb waves in heterogeneous materials requires fundamental knowledge of the behavior of Lamb waves in such materials. This basic knowledge is critical for signal processing in determining possible damage that can be detected by the propagating wave. Recently, Lamb wave methods have been used to simultaneously survey large areas of composite structures. However, such methods are more complex than traditional ultrasonic testing because Lamb waves have dispersive characteristics, namely, the wave speed varies depending on the frequency, modes and thickness of the plates. This work investigates the propagation characteristics of Lamb waves in composites, focusing on group velocity and characteristic wave curves. For symmetric laminates, a robust method is proposed by imposing boundary conditions on the mid-plane and top surface to separate symmetric and antisymmetric wave modes. The dispersive and anisotropic behavior of Lamb waves in two different types of symmetrical laminates is theoretically studied in detail. The dispersion of Lamb waves was studied for 10 symmetric and asymmetric modes. It is shown that only fundamental modes are not characterized by a cutoff frequency, which indicates the interaction of fundamental modes with composite layers in the low-frequency range.*

Key words: *Lamb wave, group velocity dispersion, wavelet analysis, laminar composites.*

Introduction.

Non-destructive testing [1, 2] and structural health monitoring [3, 4] have traditionally been the two main wavelet transform methods for assessing the integrity and degradation of composite systems widely used in construction. Implementation of an active diagnostic procedure that uses ultrasonic waves to detect damage, localize and subsequently evaluate damage involves understanding the propagation characteristics of these waves in composites.

Factors that influence the speed of wave mode propagation include the laminate laying features, wave direction, frequency and interface conditions. The dependence of the wave front speed on frequency leads to the need for a detailed study of the dispersion properties of directed waves propagating along the plane of an elastic composite plate with boundaries free from mechanical stress (Lamb waves).

As a rule, the direction of waves in laminar composites is classified by polarization perpendicular to the composite plate (symmetric S waves, antisymmetric

A waves) and parallel to the plate (shear horizontal SH waves).

For waves propagating in multilayer composites, wave interactions depend on the properties of the constituents, geometry, direction of propagation, frequency, and interfacial conditions. If the wavelengths significantly exceed the dimensions of the constituent composites (the diameters of the fibers and the distance between them), each plate can be considered as an equivalent homogeneous orthotropic or transversally isotropic material with an axis of symmetry parallel to the fibers. The study of Lamb waves (wavelet analysis) in composites [5] is most often carried out using two theoretical approaches, namely, exact solutions using three-dimensional elasticity theory and approximate solutions using plate theory.

Saito and Okabe [6] investigated the dispersion relation of Lamb waves propagating in a cross-ply CFRP laminate. Using a formalism of the multi-layer Lamb wave model, they compared a homogeneous single-layer model and multi-layer models.

Liu and Huang [7] examined the effect of inclusion shapes, inclusion contents, inclusion elastic constants, and plate thickness on the dispersion relations and modes of wave propagation in inclusion-reinforced composite plates. They determined the dispersion relations and the modal patterns of Lamb waves using the dynamic stiffness matrix method.

Orta et al. [8] introduced the new computational framework which allows to estimate the dispersion curves for the first nine symmetric and nine anti-symmetric Lamb modes. Analytically calculated dispersion curves using 5-SDT for different propagation directions and polar plots for selected frequency of different materials are compared with the results from both the semi analytical finite element method, and lower order shear deformation theories.

Ma et al. [9] constructed dispersion relations using the formulas of reverberation rays in a three-dimensional Cartesian coordinate, and numerically solved the transcendental equations using an improved mode tracking method.

Peddeti and Santhanam [10] formulated a semi-analytical finite element method (for the acoustoelastic problem of guided waves in weakly nonlinear elastic plates). It was shown that the formulation of this method provides phase velocity dispersion curve results identical to the results obtained for the problem of a plate under uniaxial and uniform tensile stress.

The character of the elastic waves causes that damage detection based on the analysis of the dynamic response of an interrogated structure becomes rather difficult [11, 12].

However, in a relatively small number of studies, dispersions of not phase, but group velocities of Lamb waves are considered [13 – 17].

So, the knowledge of moduli and group velocity dispersion enables the optimal location of the sensors in order to detect the potential damage.

The purpose of this work is to study the group velocity dispersion of symmetric and antisymmetric Lamb waves in laminar composites with different stacking structures.

Analysis of Lamb wave velocity profiles.

In general, transition waves propagating in anisotropic composites cause disturbances for all three displacement components. It is necessary to separately analyze the propagation of waves along the symmetry axes, namely, to take into account the splitting of *S*-, *A*- and *SH*-waves. The ultimate goal of the study is to compare the polynomial and exponential forms of the dispersion law for laminar composites. A Cartesian coordinate system is used in which the *z*-axis is perpendicular to the mid-plane of the composite laminate. The distance between the two outer surfaces of the laminate is z = ± δ /2. Let us consider the case of propagation of a packet of Lamb waves in the direction of δ. Each layer of the composite laminate is considered as a monoclinic material with a plane of symmetry (x-y). The relationship between mechanical stress and deformation takes the following matrix form

$$A_i = G_{ki} D_k , \tag{1}$$

where *A* and *D* are the coefficients of matrices; *G* is the stiffness matrix.

Lamb waves can be considered as standing waves in the *z*-direction of the plate. The result of this assumption is a model of wave motion in the form of a superposition of plane harmonic waves. Each plane harmonic wave moving in the ***k*** direction is represented by displacement coefficients

$$\{\alpha_1,\alpha_2,\alpha_3\}=\{\beta_1(z),\beta_2(z),\beta_3(z)\}\exp\left[i\left(k_x x+k_y y-\omega t\right)\right] , \qquad (2)$$

where k = $[k_x, k_y]^T$ and its magnitude $k = |\boldsymbol{k}| = \omega / \upsilon_p = 2\pi/\lambda$ is the wave number; ω is the angular frequency; λ is the wavelength and υ_p is the phase velocity. In the x-y plane, $\boldsymbol{k} = k\,[\cos\eta, \sin\eta]^T$, where η is the direction of wave propagation.

In an off-axis laminar composite plate, solutions to the equation of motion can be simply separated into symmetric and antisymmetric waves. This consideration allows us to write down a fairly simple analytical representation

$$\beta_{1,s}=E_s\cos\mu z,\ \beta_{2,s}=F_s\cos\mu z,\ \beta_{3,s}=G_s\cos\mu z$$

$$\beta_{1,\alpha}=E_\alpha\cos\mu z,\ \beta_{2,\alpha}=F_\alpha\cos\mu z,\ \beta_{3,\alpha}=G_\alpha\cos\mu z \qquad (3)$$

where μ is the variable to be determined by Lamb wave kinematics; subscripts "*s*" and "*a*" represent symmetric and antisymmetric modes, respectively.

Substituting these equations into the equations of symmetrical wave motion, leads to an expression in matrix form

$$\begin{bmatrix}\Lambda_{11}-\rho\omega^2 & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{12} & \Lambda_{22}-\rho\omega^2 & \Lambda_{23}\\ \Lambda_{13} & \Lambda_{23} & \Lambda_{33}-\rho\omega^2\end{bmatrix}\begin{Bmatrix}E_s\\ F_s\\ G_s\end{Bmatrix}=0, \qquad (4)$$

where the overbar denotes complex conjugation.

The relationship between the elements of matrix ($\boldsymbol{\Lambda} - \rho\omega^2\boldsymbol{I}$), stiffness matrix, and 3×3 identity matrix I have a polynomial form.

Nontrivial solutions E_s, F_s and G_s in equation (3) lead to the following sixth-order polynomial in μ

$$\mu^6+e_1\mu^4+e_2\mu^2+e_3=0, \qquad (5)$$

where e_i (i = 1,2,3) are real-valued coefficients of G_{ij}, k, and $\rho\omega^2$.

Table 1 – Spectral profile of Lamb waves for laminate A1 (symmetric modes).

f'	υ'_g		f'	υ'_g		
	S_0	SH_0		S_1	S_2	SH_2
0.5	3.325	2.384	5.0	0.962	0.001	0.001
1.0	3.218	2.321	5.6	0.921	0.002	0.003
1.5	3.085	2.305	6.2	0.824	1.512	1.264
2.0	2.798	2.208	6.8	0.841	2.358	1.587
2.5	2.237	2.126	7.4	0.935	2.857	1.698
3.0	1.749	2.111	8.0	1.045	3.042	1.762
4.0	0.387	1.564	8.6	1.018	3.110	1.852
5.0	1.400	0.631	9.2	1.089	3.043	1.964
6.0	1.310	0.735	9.5	1.088	3.002	1.993
7.0	1.182	0.786	10.1	1.070	2.804	2.057
8.0	1.087	0.811	10.8	0.993	2.220	2.125
9.0	1.010	0.832	11.4	0.968	0.995	2.173
10.0	1.000	0.846	12.0	0.970	0.484	2.186

Table 2 – Spectral profile of Lamb waves for laminate A1 (asymmetric modes).

f'	υ'_g	f'	υ'_g	f'	υ'_g		
	A_0		A_1		A_2	A_3	SH_3
0.5	0.651	2.5	1.882	8.5	0.593	0.003	0.001
1.0	0.847	2.9	2.456	8.6	0.612	0.227	0.001
1.5	0.851	3.3	2.614	8.7	0.715	0.418	0.002
2.0	0.856	3.7	2.913	8.8	0.783	0.623	0.003
2.5	0.623	4.1	3.111	8.9	0.805	0.701	0.003
3.0	0.678	4.5	3.152	9.0	0.890	0.862	0.004
3.5	0.699	4.9	3.112	9.1	0.904	0.871	0.125
4.0	0.734	5.3	3.087	9.2	0.928	0.885	0.364
4.5	0.790	5.7	2.924	9.3	0.957	0.889	0.541
5.0	0.802	6.1	2.631	9.4	0.981	0.896	0.683
5.5	0.813	6.5	2.185	9.5	1.061	0.900	0.754
6.0	0.845	6.9	1.598	9.6	1.082	0.882	0.974
6.5	0.887	7.3	1.273	9.7	1.106	0.874	1.116
7.0	0.902	7.7	0.832	9.8	1.125	0.856	1.277
7.5	0.883	8.1	0.401	9.9	1.143	0.830	1.452
8.0	0.879	8.5	0.368	10.0	1.162	0.795	1.5833
8.5	0.872	8.9	0.420	10.1	1.175	0.791	1.986
9.0	0.870	9.3	0.468	10.2	1.188	0.784	2.178
9.5	0.868	9.7	0.502	10.3	1.205	0.781	2.376
10.0	0.867	10.2	0.539	10.7	1.203	0.791	2.715

The simplified Lamb wave propagation model assumes ideal coupling between layers of the laminated composite in the z-direction. Accounting for laminate heterogeneity requires an exponential change in the displacement components

$$\beta_1 = E\exp(i\mu z),\ \beta_1 = F\exp(i\mu z),\ \beta_1 = -G\exp(i\mu z). \tag{6}$$

Table 3 – Spectral profile of Lamb waves for laminate A2 (asymmetric modes).

f'	υ'_g	f'	υ'_g	f'	υ'_g		
	A_0		A_1		A_2	A_3	SH_3
0.5	0.898	2.5	1.218	8.5	0.924	0.003	0.002
1.0	0.898	2.9	1.530	8.6	0.895	0.008	0.164
1.5	0.897	3.3	1.809	8.7	0.861	0.012	0.308
2.0	0.897	3.7	2.184	8.8	0.837	0.016	0.407
2.5	0.897	4.1	2.394	8.9	0.820	0.021	0.593
3.0	0.896	4.5	2.426	9.0	0.815	0.028	0.699
3.5	0.895	4.9	2.385	9.1	0.793	0.089	0.715
4.0	0.894	5.3	2.288	9.2	0.765	0.187	0.805
4.5	0.894	5.7	2.235	9.3	0.737	0.352	0.881
5.0	0.893	6.1	1.980	9.4	0.718	0.605	0.973
5.5	0.893	6.5	1.684	9.5	0.694	0.831	1.113
6.0	0.893	6.9	1.295	9.6	0.711	0.927	1.188
6.5	0.892	7.3	1.064	9.7	0.725	1.164	1.246
7.0	0.892	7.7	0.845	9.8	0.740	1.235	1.358
7.5	0.891	8.1	0.555	9.9	0.756	1.380	1.455
8.0	0.891	8.5	0.485	10.0	0.768	1.486	1.557
8.5	0.891	8.9	0.316	10.1	0.773	1.604	1.618
9.0	0.890	9.3	0.484	10.2	0.791	1.728	1.735
9.5	0.890	9.7	0.587	10.3	0.804	1.872	1.882
10.0	0.890	10.2	0.615	10.7	0.809	1.914	1.912

Spectral dependences of the dimensionless group velocity $\upsilon'_g = \upsilon_g/\upsilon_T$ for fixed values of the dimensionless frequency $f' = \omega\delta/\upsilon_T$ along the θ direction of laminates A_1 and A_2 are given in Tables 1 - 4. The value υ_T defined as $(G_{12}/\rho)^{0.5}$ is the transverse wave velocity in lamina (associated with shear in plane).

Summary and conclusions.

Exact solutions of Lamb waves in a plate can be established on the basis of three-dimensional elasticity theory and subsequently extended to a laminate with an arbitrary structure. For symmetrical laminates, a reliable wave mode separation method is used. A numerical method for obtaining group velocity dispersions and wave curves is proposed. The dispersions and characteristic wave curves of Lamb waves are analyzed for two types of laminates. It was found that the A0 mode has the best characteristics for structural monitoring of laminar composites. The speed of propagation of multi-frequency components within the wave packet remains almost unchanged, which causes only slight deformation of the wave packet shape when moving in the composite layers. In addition, the significantly low attenuation of A_0 mode and high sensitivity to the growth of delamination in the sample indicate the practical value of using symmetric modes as a diagnostic tool.

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