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WAVELET MODELING OF HIGH-FREQUENCY WAVE PROPAGATION IN ANISOTROPIC COMPOSITES

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Abstract. *This paper presents a wavelet model for studying high-frequency wave propagation in thin and moderately thick anisotropic laminated composites. A spectral finite element is developed that can describe the propagation of axial and shear waves.*
Key words: *composite, wave propagation, wavelet, finite element.*

The need to predict transient responses, characterize mechanical properties, and perform non-destructive evaluation of materials can be effectively addressed by studying wave propagation characteristics of elastic structures [1]. Composite (elastic) structures are increasingly used in many industries due to several advantages, including higher specific strength and modulus, fewer joints, improved fatigue life, and higher corrosion resistance. In particular, Lamb wave-based structural health monitoring, which aims to perform non-destructive assessment, is a very active area of research.

Analytical solutions for wave propagation are not available for most practical structures due to the complex nature of the governing differential equations and boundary/initial conditions. The finite element method is the most popular numerical method for modeling wave propagation phenomena [2]. However, for accurate predictions using the finite element method, typically 20 elements must span a wavelength, which results in a very large system size and huge computational costs for analyzing wave propagation at high frequencies. In addition, the solution of inverse problems is very difficult using the finite element method [3]. The spectral finite element, which follows the procedure of modeling the finite element method in the transformed frequency domain, is very suitable for the analysis of wave propagation. The frequency-domain formulation of the spectral finite element provides a direct connection between the output and the input through the transfer function of the system (frequency response function) [4]. The spectral finite element has very high computational efficiency because the nodal displacements are related to the nodal tractions through a stiffness matrix dependent on the frequency and number of waves. The mass distribution is accurately fixed, and an accurate elemental dynamic stiffness matrix is derived.

In this paper, a new two-dimensional wavelet based spectral finite element framework for high-frequency analysis with finite dimensions and anisotropic properties of composite materials is presented. The method derives the governing partial differential equations for wave motion and their time approximation using compactly supported high-order Daubechies scaling functions. One of the objectives of the work is to perform eigenvalue analysis to separate the given partial differential equations in spatial dimensions. In the following, the separated partial differential equations are approximated in one spatial dimension using Daubechies low-order scaling functions followed by an eigenvalue analysis similar to the time approximation. The resulting ordinary differential equations are solved exactly in the frequency-wave domain and the solution is used as a shape function for the two-dimensional spectral element.

The working frame in the computational model is a laminated composite plate of thickness h with the global coordinate origin at the mid-plane of the plate and the *Z*-axis perpendicular to the mid-plane. The results of the spectral finite element Fourier method showed that the governing partial differential equations for wave propagation in laminated composites have five degrees of freedom: u, υ, w, φ, and ψ. The terms $u\,(x, y, t)$ and $\upsilon\,(x, y, t)$ are the mid-plane ($z = 0$) displacements along the *X* and *Y* axes; $w\,(x, y, t)$ is the transverse displacement in the *Z* direction, and $\varphi\,(x, y, t)$ and $\psi\,(x, y, t)$ are the rotational displacements about the *X* and *Y* axes, respectively. The quantities N_{xx}, N_{xy}, N_{yy} are in-plane force resultants, M_{xx}, M_{xy}, M_{yy} are moment resultants, and Q_x, Q_y denote the transverse force resultants. Stiffness constants A_{ij}, B_{ij}, D_{ij} are defined as

$$\left[A_{ij}, B_{ij}, D_{ij}\right] = \sum_{q=1}^{N_p} \int_{z_q}^{zq+1} Q'_{ij}\left[1, z, z^2\right] dz = \int_{-h/2}^{+h/2} \left\{1, z, z^2\right\} \rho\, dz\,, \quad (1)$$

where Q'_{ij} are the stiffnesses of the *q*-th lamina in coordinate system, N_{p} is the total number of plies, ρ is the mass density.

The function $u\,(x, y, z)$ can be approximated at an arbitrary scale as

$$u(x, y, z) = \sum_k u_k(x, y)\varphi(\tau - k),\ k \subset \mathbf{Z}\,, \quad (2)$$

where $u_{\mathrm{k}}\,(x, y)$ are the approximation coefficients at a certain spatial dimension (x and y) and $\varphi\,(\tau)$ are scaling functions associated with Daubechies wavelets.

The next step after determining the shear coefficients lying outside the local volumes of the laminated composite, through the internal coefficients taking into account the periodic expansion, can be written in the form of a matrix equation

$$Aij\left(\frac{d^2u_{ij}}{dx^2}\right)+A_{rp}\Lambda_f\left(\frac{d\upsilon_p}{dx}\right)=I_f\gamma_r{}^2+I_f\gamma_r\,, \qquad (3)$$

where Λ_f are the first-order connection coefficient matrix elements obtained after periodic extension; I_f are the inertial coefficients; are the eigenvalues of the shift matrix.

Finally, the transformed nodal forces $\{F^e\}$ and transformed nodal displacements $\{u^e\}$ are related by

$$\{F^e\}=[K^e]\{u^e\}\,, \qquad (4)$$

where $[K^e]$ is the exact elemental dynamic stiffness matrix.

The computational model allowed to obtain dispersion relations for the 8-layer $[0]_8$ laminate. The first antisymmetric mode (*A0*) corresponded to the frequency range below 30 kHz. For high frequencies exceeding 2000 kHz, the phase and group velocities increased according to a polynomial law. In addition, the spectral method of Fourier transforms predicted the appearance of the second antisymmetric mode (*A1*) and the first shear mode (*SH*1) for the same frequency range. Spectral relationships for composite laminates with an asymmetric ply sequence $[0_5/90_5]$ indicate that their wavenumbers have significant real and imaginary parts, implying their non-uniform nature (attenuation during propagation). It was found that fixed boundary reflection from the internal surfaces of the composite sample preserves the shape of the excited wave packet.

The wavenumber values correspond to a sampling time of Δt = 2 μs, which gives a Nyquist frequency of f_n = 250 kHz. The wavelet transform method predicts accurate wavenumbers only up to a certain fraction of the Nyquist frequency. This fraction depends only on the order of the Daubechies scaling function used to approximate the original partial differential equations in time.

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