

### Секція 3. Технічні науки

*Alexander Pysarenko, associate professor, PhD,  
Odessa State Academy of Civil Engineering and Architecture  
ORCID: 0000-0001-5938-4107*

#### **SHEAR WAVE PROPAGATION IN LAMINATED COMPOSITE**

Internet address of the article on web-site:

<http://www.konferenciaonline.org.ua/ua/article/id-2003/>

Prediction of transient responses, characterization of mechanical properties and non-destructive evaluation is achieved by analyzing wave propagation in elastic composite structures. Laminar composites are widely used in many industries, including transportation, wind energy due to a number of advantages. These advantages include higher specific strength and modulus, fewer joints, improved fatigue life and higher corrosion resistance. Lamb wave-based structural health monitoring allows for non-destructive evaluation using integrated actuators and sensors [1]. Having a physical model of wave propagation in combination with experimental measurements is a prerequisite for a complete characterization (presence, location and severity) of damage.

Modeling shear wave propagation in composites is significantly more challenging than modeling for isotropic structures [2, 3]. Analytical solutions for wave propagation are not available for most practical laminar composite structures due to the complex nature of the governing differential equations and boundary/initial conditions. The finite element method is the most popular numerical method for modeling wave propagation phenomena [4]. However, for accurate predictions using the finite element method, usually a significant number of elements must span a wavelength, which leads to a very large system size and huge computational costs for wave propagation analysis at high frequencies. The spectral finite element method, which follows the transformed frequency domain finite element modeling procedure, is very suitable for wave propagation analysis. The frequency domain formulation of the spectral finite element provides a direct relationship between the output and the input through the system transfer function (frequency response function). The spectral finite element has very high computational efficiency because the nodal displacements are related to the nodal tractions via a frequency-wave stiffness matrix dependent on the number of waves. The mass distribution is accurately captured and an accurate elemental dynamic stiffness matrix is derived.

In this paper, a 2-D wavelet finite element technique based on first-order shear strain theory is advanced for high-frequency analysis of finite-size waveguides with anisotropic material properties. The governing partial differential equations for the wave motion and their time approximation using high-order Daubechies scaling functions with compact support are presented. An eigenvalue analysis is performed to separate the reduced partial differential equations in the spatial dimensions. The separated partial differential equations are then approximated in one spatial

dimension using Daubechies low-order scaling functions, followed by an eigenvalue analysis similar to the time approximation. The resulting ordinary differential equations are solved exactly in the frequency-wavenumber domain and the solution is used as a shape function for a 2-D spectral element. Numerical relationships between first-order shear deformation theory and classical laminated plate theory in dispersion curves provide spectral relationships and represent the time-domain responses. The results for the new wavelet-based spectral finite element formulation are validated by simulations using shear flexible shell elements.

The improvement of the shear wave propagation model was based on the modification of the governing differential equations for wave propagation. For this purpose, a laminated composite plate of fixed thickness with the global coordinate system origin in the mid-plane of the plate and a normal axis that is perpendicular to the mid-plane was chosen as the working model. In particular, the first-order shear deformation model leads to governing differential equations in partial derivatives for wave propagation that have five degrees of freedom.

Without loss of generality in all essential aspects of the problem, a laminate consisting of an arbitrary number of orthotropic layers such that the axes of symmetry of the material are parallel to the lateral surfaces of the laminated composite plate. The time approximation of the governing partial differential equations and the boundary conditions have three independent spatial variables and their derivatives, which makes them very complex to solve. Therefore, compactly supported Daubechies scaling functions are used to approximate the time variable. This procedure reduces the set of equations to partial differential equations with only two spatial variables associated with the side surface of the composite specimen. Compactly supported scaling functions have only a finite number of filter coefficients with nonzero values, which allows for easy handling of finite geometries and imposing boundary conditions.

Numerical calculations indicate that each shear wave mode corresponds to a corresponding degree of freedom, which is present in the governing equations based on the first-order deformation theory. The mode numbers represent the cutoff frequencies for each degree of freedom, where the wave numbers change from imaginary to real. These cutoff frequencies record the progression of the shear wave modes through the local volume of the laminated composite. Comparisons of the group and phase velocity dispersion for the fundamental antisymmetric shear wave mode revealed a large discrepancy at high frequencies, especially in the case of composite laminates, which have a lower transverse shear modulus (compared to isotropic materials).

### **References:**

1. Mitra M., and Gopalakrishnan S. Guided wave based structural health monitoring. A review. *Smart Materials and Structures*. 2016. Vol. 25. P. 053001. <https://doi.org/10.1088/0964-1726/25/5/053001>
2. Maio L. et al. Ultrasonic wave propagation in composite laminates by numerical simulation. *Composite Structures*. 2015. Vol. 121. Pp. 64-74. <https://doi.org/10.1016/j.compstruct.2014.10.014>

3. Kudela P., Radzienski M, and Ostachowicz W. Wave propagation modelling in composites reinforced by randomly oriented fibers. Journal of Sound and Vibration. 2018. Vol. 414. Pp. 110-125. <https://doi.org/10.1016/j.jsv.2017.11.015>
4. Willberg C. et al. Comparison of different higher order finite element schemes for the simulation of Lamb waves. Computer methods in applied mechanics and engineering. 2012. Vol. 241. Pp. 246-261. <https://doi.org/10.1016/j.cma.2012.06.011>

*Корбан Віктор Харитонович, доктор технічних наук,  
доцент, кафедра технічної експлуатації флоту,  
Національний університет «Одеська морська академія», м. Одеса*

## ОЦІНКА НАДІЙНОСТІ ФУНКЦІОНУВАННЯ СУДНОВОЇ ЕНЕРГЕТИЧНОЇ УСТАНОВКИ

Інтернет-адреса публікації на сайті:  
<http://www.konferenciaonline.org.ua/ua/article/id-2007/>

Надійність суднової енергетичної установки (СЕУ), що складається з  $n$ -систем із номерами « $i$ », визначається тільки випадковими, неконтрольованими факторами, тобто моментом виходу з ладу її систем, яка може перебувати тільки в одному з двох станів: працездатному або непрацездатному. Будемо вважати, що її системи (паливна, система змащення, система охолодження тощо) з'єднані послідовно. Непрацездатність (вихід з ладу) однієї системи тягне за собою вихід з ладу всієї СЕУ. Позначимо через  $t_i$  – момент виходу з ладу  $i$ -тої системи. Тоді  $W$  критерій ефективності СЕУ, який показує, що до моменту  $t$  СЕУ перебувала в працездатному стані або в  $[0, t]$  втратила працездатність, можна записати у вигляді:

$$\begin{aligned} W(t) &= 1, \text{ при } t < \min[t_i], \text{ а } 1 \leq i \leq n; \\ W(t) &= 0, \text{ при } t \geq \min[t_i], \text{ а } 1 \leq i \leq n \end{aligned} \quad (1)$$

де 1 – означає працездатність СЕУ.

Як критерій можна використовувати і сам час  $T$  безвідмовної роботи СЕУ, який дорівнює:

$$T = \min[t_i], \text{ при } 1 \leq i \leq n. \quad (2)$$

Величини  $t_i$  є випадковими із законами розподілу  $P_i(t)$ , що дають імовірність виходу певної системи СЕУ з ладу до моменту  $t$ . Таким чином, присутні неконтрольовані випадкові фактори. Однак, стратегія поки тільки одна, оскільки конструкція СЕУ повністю задана. Підвищення надійності, тобто працездатності СЕУ до даного моменту  $t_0$  або збільшення часу роботи  $T$ , може бути досягнуто за рахунок дублювання в кожній системі її агрегатів, якщо