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APPLICATION OF HAAR-WAVELET METHOD TO LAYERED STRUCTURES

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High-performance composites exhibit a number of exceptional mechanical, thermal and electrical properties. Structural applications of laminated composites can be used in various industrial fields that require a material with high strength, stiffness and low weight [1]. It is worth noting a number of experimental and analytical studies of the mechanical properties of such laminated structures. It was found that the strength, stiffness, high Young's modulus and tensile strength of reinforced composites can be significantly improved by minor additives that modify their laminated structure. The non-uniform distribution of the reinforcement phase allows to classify reinforced composites with a modified layered structure as functionally graded materials [2]. Numerous studies of the mechanical properties of functionally graded composites in the form of beams, plates and shells were primarily focused on the analysis of their behavior under static bending, elastic loss of stability, as well as linear and nonlinear characteristics of free vibration.

Wavelet transform methods have been recently considered as a promising tool for analyzing mechanical properties and heterogeneities of layered composites under fixed boundary conditions [3]. Various types of wavelets have been used in numerical approximations. However, among them, the Haar wavelet has attracted wide attention due to its unique properties such as easy applicability, orthogonality, and compact support. The Haar wavelet discretization method has been successfully adopted for analyzing a wide range of problems such as: solid mechanics, free vibrations of composite layered cylindrical, conical, ring-plate structures, and vibrations of axially functionally graded beams with various boundary conditions [4].

In this paper, the efficiency of using the Haar wavelet method to predict the nonlinear vibration behavior of functionally graded materials is analyzed.

A beam fixed on a three-parameter nonlinear elastic foundation with cubic nonlinearity and a shear layer was considered as a model. Based on the first-order shear deformation theory in combination with the von Karman nonlinearity, the nonlinear governing equations of the beams made of carbon fiber reinforced composites are derived using the Hamilton principle. The resulting governing equations and the corresponding boundary conditions are first discretized into nonlinear algebraic equations using the Haar wavelet discretization method, and then solved by the direct iteration method to obtain the linear and nonlinear frequencies of the functionally graded beams.

The load-displacement ratio of the working model considered in this methodology is given by

$$F_0 = k_L \omega + k_N \omega^3 - k_S \frac{\partial^2 \omega}{\partial x^2}, \quad (1)$$

where F_0 is the reaction force of the nonlinear elastic foundation; k_L , k_N , and k_S are the nonlinear coefficients of the Winkler elastic foundation; ω is the first displacement component.

Nonlinear von Karman type strain-displacement relations can be written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z \frac{\partial \varphi}{\partial x} + \frac{1}{2} \left(\frac{\partial \omega}{\partial x} \right)^2, \quad \gamma_{xz} = \frac{\partial \omega}{\partial x} + \varphi \quad (2)$$

where u is the second displacement component; φ denotes the rotation of the beam cross section; γ is the strain component.

Solving differential equations using the Haar wavelet discretization method requires integrals of the Haar wavelet functions p_n for ξ argument, which can be calculated using the relation

$$p_n(\xi) = \frac{\xi^n}{n!}, \quad (3)$$

where n is the integral order of the Haar wavelet function.

Summary and conclusions. The nonlinear vibration behavior of functionally graded laminated composite beams supported on a nonlinear elastic foundation in a thermal environment was investigated in this study using the Haar wavelet discretization method. The first-order shear deformation theory together with the von Karma nonlinearity was adopted to model the kinematic relations. The initial thermal stress of the material properties was considered in the theoretical

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modeling. The system of nonlinear governing equations of the functionally graded laminated structures on a nonlinear elastic foundation was derived and then solved using the Haar wavelet discretization method combined with the direct iteration method to obtain the linear and nonlinear natural frequencies. The analysis of the proposed model showed that the linear fundamental frequency increases and the nonlinear frequency ratio decreases with an increase in the linear or shear stiffness coefficient of the foundation.

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