DOI 10.36074/logos-09.05.2025.078

MULTISCALE WAVELET ANALYSIS IN POROUS MEDIA

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An active area of experimental and theoretical research is to determine the relationship between microstructure and material properties such as mechanical, electrical, etc. [1]. Numerous studies have led to a better understanding of issues such as deformation, fracture, strength, electrical conductivity. However, it has been recognized that several open questions in this area could benefit significantly from recognizing that material heterogeneity (microstructure) is scale dependent, i.e. it is multiscale for (most) materials. Describing the relationship of material properties at different scales is quite a complex task [2]. Traditionally, problems of a multiscale nature, in particular the description of the porosity of inhomogeneous materials, are dealt with by different models using wavelet transforms. The composite wavelet matrix method consists of an approach in which different computational methodologies (molecular dynamics simulations, Monte Carlo simulations, and Potts Q-states) are applied to a region of a material simultaneously at coarse and fine spatial scales [3, 4].

Wavelet coefficient matrices are derived from energy maps representing the spatial distribution of local excess energy in microstructures. A complete description of the material is obtained by merging the wavelet coefficient matrices representing the material at different scales through a composite wavelet matrix. The composite wavelet matrix then characterizes the material over a range of different scales. This study complements the composite wavelet matrix method, since it deals with the definition of specific measures for comparing information at different scales. In addition, it is applicable to a general type of available data, such as from experiments, modeling, etc. Although the process is detailed in relation to multiscale porosity, it can be generally extended to various forms of material heterogeneity. The geometry of a porous medium can be described by a fundamental function, which is defined as one for spatial positions in the matrix and zero for positions in defects. The specific area of defects is defined as

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$$S = \lim_{\Delta V \to 0} (\Delta A / \Delta V) = -4q(1-q)\partial\rho(r)/\partial r\,, \tag{1}$$

where ΔV is the infinitesimal volume; ΔA is the portion of the solid–flaw interface crossing ΔV; ρ is the autocorrelation function; q is the porosity; r is the spatial distance between two points.

The wavelet ψ (x) transforms the function f(x) according to the formula

$$W_f(a,b) = \int_{R^D} f(x)\psi(x)dx. \tag{2}$$

The two-parameter family of functions

$$\psi_{a,b}(x) = a^{-D/2}\psi\left(\frac{x-b}{a}\right), \tag{3}$$

obtained from a single family, ψ, called the mother wavelet, by expansion by a factor a and translation by a factor b. Here D denotes the spatial dimension of the problem and $a \in R_+$; $a \neq 0$; and b, $x \in R^D$.

Discrete wavelet analysis based on orthogonal decomposition of the signal can be performed using fast algorithms. Given a wavelet transform $W_f(a, b)$, associated with a function f, it is possible to reconstruct f and/or construct its representation in a range of multi-scales between s_1 and s_2 ($s_1 \leq s_2$)

$$f_{s_1,s_2}(x) = c_\psi^{-1}\int_{s_1}^{s_2}\int_{R^D} W_f(a,b)\psi_{a,b}(x)db\left(a^{1+D}\right)da. \tag{4}$$

By setting $s_1 \to 0$; $s_2 \to 1$; function f may be reconstructed.

The magnitude of the variance $W_f(a, b)$ can be used to estimate the energy of the wavelet transform as a function of the scale a. In this case, the estimate in Fourier space allows us to write the following relation

$$\sigma_W^2(a) = \left\langle [W_f(a,b)]^2 \right\rangle a^D (2\pi)^{-1}\int_{R^D} P_f(k)[\hat{\psi}(ak)]^2 dk, \tag{5}$$

where $P_f(k)$ is the power spectrum of f.

As a special case, we can consider the influence of porosity on the representative property, i.e. mechanical failure. The ratio of the macroscopic (nominal) failure stress σ_c to the failure stress of the background homogeneous system (matrix material without pores), σ_0 is expressed as

$$\sigma_c\sigma_0^{-1} = \left[1 + k\left(-\frac{\ln V}{\ln q}\right)^{\alpha}\right]^{-1}, \tag{6}$$

where q is the volume fraction of voids; $1/[2(D-1)] \leq \alpha \leq 1$.

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Summary and conclusions. The general process of information binding is considered in this calculation method and is studied in detail with respect to statistically stationary and isotropic porous media. An example of application is mechanical failure; however, the process is general enough and can be extended to any properties where scale hierarchy is important. The definition of a dominant scale or range of scales is a natural consequence of the multiscale description.

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