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SCATTERING OF ANTI-SYMMETRIC MODES OF LAMB WAVES BY DEFECTS

Pysarenko Oleksandr Mykolayovych[1]

1. candidate of physical and mathematical sciences,
associate professor of the department of physics
Odessa State Academy of Civil Engineering and Architecture, UKRAINE
ORCID ID: 0000-0001-5938-4107

Lamb waves are a class of guided elastic waves that propagate in thin plate-like structures. These waves can be decomposed into symmetric (S) and anti-symmetric (A) modes based on their displacement characteristics relative to the plate's mid-plane. Among them, the fundamental anti-symmetric mode (A0) plays a crucial role in non-destructive evaluation due to its high sensitivity to surface and near-surface anomalies.

When Lamb waves encounter material discontinuities - such as cracks, delaminations, or voids - they scatter, causing a redistribution of wave energy. This scattering may involve reflection, transmission, and conversion into other wave modes. The interaction of A0 modes with defects is of particular interest because of the low group velocity and dominant out-of-plane displacement of this mode, which make it especially effective in detecting defects close to the surface [1].

The propagation of Lamb waves in isotropic plates is governed by the elastodynamic wave equation. Under plane strain conditions, the displacement field can be expressed using scalar and vector potentials. Applying boundary conditions for stress-free surfaces leads to the characteristic Lamb wave dispersion equations. For the anti-symmetric modes, the dispersion relation takes the form:

$$tg(qh) = -\frac{4k^2 pq}{\left(q^2 - k^2\right)^2}, \tag{1}$$

where k is the Lamb wave number; h is half the plate thickness; p and q are the functions of the angular frequency ω and the longitudinal and shear wave velocities, respectively.

When an A0 wave interacts with a defect, the scattered field can be described in cylindrical coordinates as a summation of outgoing wave components:

$$u(r,\theta) = \sum_{n=-\infty}^{\infty} A_n H_n^{(1)}(kr)\exp(in\theta), \quad (2)$$

where $H_n^{(1)}$ is the Hankel function of the first kind, representing outgoing cylindrical waves; A_n are the modal scattering coefficients; (r, θ) denote the polar coordinates centered at the defect.

These coefficients encapsulate the effect of the defect on the scattered field and depend on its size, shape, orientation, and material contrast [2].

Consider the specific case of a circular void embedded in a plate. The interaction of the A0 mode with such a defect causes the wave to scatter in all directions. The resulting field can be expressed by expanding the incident and scattered waves into cylindrical harmonics and applying appropriate boundary conditions. For such problems, analytical techniques based on eigenfunction expansion are used to derive closed-form expressions for the scattering coefficients.

For numerical modeling, finite element methods (FEM) are commonly used to simulate A0 scattering from complex defect geometries. The incident wave is typically introduced using a tone-burst or harmonic excitation, and the resulting wave field is analyzed to extract reflected, transmitted, and mode-converted components. The total displacement field can then be decomposed into modal contributions using techniques such as mode filtering or wavenumber analysis.

The scattering of A0 modes is strongly frequency-dependent. At low frequencies (where the wavelength is large compared to defect size), the scattering is weak and predominantly in the forward direction. As frequency increases, stronger reflections and more pronounced mode conversions occur. This behavior is useful for characterizing defects based on frequency-dependent scattering patterns.

The amplitude and angular distribution of the scattered A0 mode contain information about the defect's geometry. In particular, sharp-edged defects like cracks cause directional backscattering, while rounded inclusions lead to smoother, more isotropic scattering. This distinction can be used in inverse algorithms for defect reconstruction.

The study of A0 mode scattering by defects provides valuable insight into wave-defect interactions and supports the development of advanced structural health monitoring (SHM) and non-destructive testing (NDT) systems. Accurate modeling of these interactions improves defect detection resolution and enables the classification of defect types based on their scattering signatures.

Summary and conclusions. The scattering behavior of anti-symmetric Lamb wave modes, particularly the A0 mode, provides significant insights into the detection and characterization of structural defects in plate-like materials. The strong sensitivity of A0 modes to surface and near-surface discontinuities makes them especially effective for non-destructive evaluation applications. This study demonstrates that the scattering pattern, including amplitude, directionality, and mode conversion, depends strongly on the defect geometry, size, and the frequency of the incident wave. Analytical and numerical models reveal that A0 mode interactions with cracks produce pronounced backscattering, while voids and inclusions lead to more isotropic scattering responses. Understanding these scattering mechanisms is essential for improving the accuracy of damage localization and classification algorithms. Future work should focus on multi-mode analysis, inverse reconstruction techniques, and the integration of experimental data to validate and refine theoretical predictions for complex real-world structures.

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