Evolution of perturbed rotations of an asymmetric Gyro in a gravitational field and a resisting medium

Article *in* Mechanics of Solids · July 2016

DOI: 10.3103/S002565441604004X

Leonid D Akulenko
Russian Academy of Sciences

Dmytro Leshchenko
Odessa State Academy of Civil Engineering and…

Alla Leonidovna
Odessa National University

ISSN 0025-6544, Mechanics of Solids, 2016, Vol. 51, No. 4, pp. 406–414. © Allerton Press, Inc., 2016.
Original Russian Text © L.D. Akulenko, D.D. Leshchenko, A.L. Rachinskaya, Yu.S. Shchetinina, 2016, published in Izvestiya Akademii Nauk, Mekhanika Tverdogo Tela, 2016, No. 4, pp. 43–52.

Evolution of Perturbed Rotations of an Asymmetric Gyro in a Gravitational Field and a Resisting Medium

L. D. Akulenko[1*], D. D. Leshchenko[2], A. L. Rachinskaya[3], and Yu. S. Shchetinina[3***]**

[1]Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, pr. Vernadskogo 101, str. 1, Moscow, 119526 Russia
[2]Odessa State Academy of Civil Engineering and Architecture, ul. Didrikhsona 4, Odessa, 65029 Ukraine
[3]Mechnikov Odessa National University, ul. Dvoryanskaya 2, Odessa, 65023 Ukraine

Received June 25, 2014

Abstract—We study the fast rotational motion of a dynamically asymmetric satellite with a spherical cavity filled with a highly viscous liquid about the center of mass under the action of gravitational torque and medium drag torques. The system obtained by averaging over the Euler–Poinsot motion and by using a modified averaging method is analyzed. An analytic study and numerical analysis are carried out.

DOI: 10.3103/S002565441604004X

Keywords: *satellite, gravitational torque, medium resistance, cavity filled with a viscous liquid, averaging.*

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Rotational motions are considered in the framework of a dynamic model of a rigid body whose center of mass moves in a circular orbit around the Earth. Dynamic problems generalized and complicated by taking into account various perturbations are still very topical. The studies in [1–18] of rotational motions of bodies about a fixed point under the action of perturbing torques of various physical nature (gravitation, drag, influence of a cavity filled with a viscous liquid, etc.) are close to those presented below.

Consider the motion of a dynamically asymmetric satellite about the center of mass with the gravitational and ambient medium drag torques taken into account. The body has a cavity completely filled with a highly viscous liquid.

To solve the problem, we introduce three Cartesian reference frames with origin at the satellite center of inertia [1]. The reference frame Ox_i $(i = 1, 2, 3)$ moves translationally together with the center of inertia, the axis Ox_1 is parallel to the position vector of the orbit perigee, the axis Ox_2 is parallel to the velocity vector of the satellite center of mass at perigee, and the axis Ox_3 is parallel to the normal to the orbit plane. The position of the angular momentum vector $\mathbf{G}$ with respect to the center of mass in the reference frame Ox_i is determined by the angles λ and δ [2]. The reference frame Oy_i $(i = 1, 2, 3)$ is related to the angular momentum vector $\mathbf{G}$ as follows: the axis Oy_1 lies in the plane Ox_3y_3 and is directed so that the vectors $\mathbf{y}_i$ $(i = 1, 2, 3)$ form a right triple [1], the axis Oy_2 lies in the orbit plane (i.e., in the plane Ox_1x_2), and the axis Oy_3 is directed along the angular momentum vector $\mathbf{G}$.

The axes of the reference frame Oz_i $(i = 1, 2, 3)$ are related to the principal central axes of inertia of the rigid body. The mutual position of the principal central axes of inertia and of the axes Oy_i is determined by the Euler angles. Then the direction cosines α_{ij} of the axes z_i with respect to the system Oy_i are expressed via the Euler angles φ, ψ, and θ by well-known formulas [1].

*e-mail: kumak@ipmnet.ru
**e-mail: leshchenko_d@ukr.net
***e-mail: powtampik@gmail.com

The equations of motion of the body with respect to the center of mass have the form [2]

$$\begin{aligned}
&\frac{dG}{dt} = L_3, \quad \frac{d\delta}{dt} = \frac{L_1}{G}, \quad \frac{d\lambda}{dt} = \frac{L_2}{G\sin\delta},\\
&\frac{d\theta}{dt} = G\sin\theta\sin\varphi\cos\varphi\left(\frac{1}{A_1} - \frac{1}{A_2}\right) + \frac{L_2\cos\psi - L_1\sin\psi}{G},\\
&\frac{d\varphi}{dt} = G\cos\theta\left(\frac{1}{A_3} - \frac{\sin^2\varphi}{A_1} - \frac{\cos^2\varphi}{A_2}\right) + \frac{L_1\cos\psi + L_2\sin\psi}{G\sin\theta},\\
&\frac{d\psi}{dt} = G\left(\frac{\sin^2\varphi}{A_1} + \frac{\cos^2\varphi}{A_2}\right) - \frac{L_1\cos\psi + L_2\sin\psi}{G}\cot\theta - \frac{L_2}{G}\cot\delta.
\end{aligned} \tag{1.1}$$

Here the L_i are the torques of the applied forces about the axes Oy_i, and the A_i $(i = 1, 2, 3)$ are the principal central moments of inertia with respect to the axes Oz_i. The projections L_i in (1.1) consist of the gravitational torque L_i^g, the external drag torque L_i^r, and the torque L_i^p due to the viscous liquid forces in the cavity inside the body; i.e.,

$$L_i = L_i^g + L_i^r + L_i^p.$$

Consider a dynamically asymmetric satellite whose moments of inertia satisfy the inequalities $A_1 > A_2 > A_3$. We assume that the angular velocity ω of the satellite motion about the center of mass is significantly greater than the angular velocity ω_0 of the orbital motion; i.e., $\varepsilon = \omega_0/\omega \sin A_1\omega_0/G \ll 1$.

The dependence of the dissipative drag torque $\mathbf{L}^r$ on the angular velocity vector $\boldsymbol{\omega}$ of the body rotation is assumed to be linear; i.e., $\mathbf{L}^r = \mathbf{I}\boldsymbol{\omega}$, where the tensor $\mathbf{I}$ has constant components I_{ij} in the body-fixed frame Oz_i [1, 6]. The medium resistance is assumed to be weak of the order of ε^2; i.e., $\|\mathbf{I}\|/G_0 \sin\varepsilon^2 \ll 1$ [3], where $\|\mathbf{I}\|$ is the norm of the drag coefficient matrix and G_0 is the satellite angular momentum at the initial time.

The satellite orbit is assumed to be circular, and hence we can assume that the atmosphere density is constant during the motion. The true anomaly ν depends on time t as follows:

$$\nu = \omega_0 t, \quad \omega_0 = \frac{2\pi}{Q}, \tag{1.2}$$

where ω_0 is the angular velocity of the orbital motion and Q is the period of revolution.

We write out the projections of the gravitational torque L_1^g and the external drag torque L_1^r onto the axis Oy_1 in the form accepted in [2, 3]. Here we present the projections on the axis Oy_1 (the projections on the other axes are similar):

$$L_1^g = 3\omega_0^2\sum_{j=1}^{3}(\beta_2\beta_j S_{3j} - \beta_3\beta_j S_{2j}), \tag{1.3}$$

$$L_1^r = -G\sum_{i=1}^{3}\left(\frac{I_{i1}\alpha_{1i}\alpha_{31}}{A_1} + \frac{I_{i2}\alpha_{1i}\alpha_{32}}{A_2} + \frac{I_{i3}\alpha_{1i}\alpha_{33}}{A_3}\right), \quad S_{mj} = \sum_{p=1}^{3}A_p\alpha_{jp}\alpha_{mp}, \tag{1.4}$$

$$\beta_1 = \cos(\nu - \lambda)\cos\delta, \quad \beta_2 = \sin(\nu - \lambda), \quad \beta_3 = \cos(\nu - \lambda)\sin\delta.$$

According to [4], the projections of the torque L_i^p due to the viscous liquid forces in the spherical cavity of the body onto the axes Oy_i ($i = 1, 2, 3$) are determined as follows:

$$L_i^p = \frac{P}{A_1A_2A_3}\{\boldsymbol{\omega}\cdot\mathbf{B}^i + 3\omega_0^2(\mathbf{D}+\mathbf{S})\cdot\boldsymbol{\alpha}^i\}, \quad i = 1, 2, 3,$$

$$\boldsymbol{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \quad \mathbf{B}^i = \begin{pmatrix} B_1^i \\ B_2^i \\ B_3^i \end{pmatrix}, \quad \boldsymbol{\alpha}^i = \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \end{pmatrix}, \quad \alpha^* = \frac{1}{1-\alpha_{33}^2},$$

$$\mathbf{D} = \begin{pmatrix} A_2A_3(A_3-A_2)[-\gamma_{31}\gamma_{33}r + \alpha^*(F_1p_{\alpha1} + M_1p_{\alpha2})] \\ A_1A_3(A_1-A_3)[-\gamma_{32}\gamma_{33}r + \alpha^*(F_2p_{\alpha1} + M_2p_{\alpha2})] \\ (A_2-A_1)[(\gamma_{32}^2-\gamma_{31}^2)r - \alpha^*(F_3p_{\alpha1} + M_3p_{\alpha2})] \end{pmatrix},$$

$$\mathbf{F} = \begin{pmatrix} \gamma_{31}\gamma_{33}\alpha_{33} + \beta_{\alpha1}\gamma_{33} + \beta_{\alpha2}\gamma_{32} \\ \gamma_{32}\gamma_{33}\alpha_{33} + \beta_{\alpha3}\gamma_{33} + \beta_{\alpha2}\gamma_{31} \\ (\gamma_{32}^2-\gamma_{31}^2)\alpha_{33} + \beta_{\alpha3}\gamma_{32} + \beta_{\alpha1}\gamma_{31} \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} \gamma_{32}^2\alpha_{32} + \gamma_{32}(\gamma_{33}\alpha_{33} - v_3) \\ \gamma_{33}^2\alpha_{31} + \gamma_{31}(\gamma_{33}\alpha_{33} - v_3) \\ \gamma_{33}(\gamma_{32}\alpha_{31} + \gamma_{31}\alpha_{32}) \end{pmatrix}, \tag{1.5}$$

$$\mathbf{S} = \begin{pmatrix} \gamma_{31}[\gamma_{33}rA_3(A_1A_2 - A_1^2 - A_2A_3 + A_3^2) + \gamma_{32}qA_2(A_1A_3 - A_1^2 - A_2A_3 + A_2^2)] \\ \gamma_{32}[\gamma_{31}pA_1(A_3A_2 - A_2^2 - A_1A_3 + A_1^2) + \gamma_{33}rA_3(A_1A_2 - A_2^2 - A_1A_3 + A_3^2)] \\ \gamma_{33}[\gamma_{32}qA_2(A_1A_3 - A_3^2 - A_1A_2 + A_2^2) + \gamma_{31}pA_1(A_2A_3 - A_3^2 - A_1A_2 + A_1^2)] \end{pmatrix},$$

$$\gamma_{3i} = \beta_1\alpha_{1i} + \beta_2\alpha_{2i} + \beta_3\alpha_{3i}, \quad i = 1, 2, 3,$$

$$p_{\alpha1} = p\alpha_{31} + q\alpha_{32}, \quad p_{\alpha2} = p\alpha_{32} + q\alpha_{31},$$

$$v_{\alpha1} = -\alpha_{22}v_1 + \alpha_{12}v_2, \quad v_{\alpha2} = -\alpha_{23}v_1 + \alpha_{13}v_2, \quad v_{\alpha3} = -\alpha_{21}v_1 + \alpha_{11}v_2,$$

$$B_1^i = [\omega_2^2A_2(A_1-A_2)(A_2-A_3+A_1) + \omega_3^2A_3(A_1-A_3)(A_3-A_2+A_1)]\alpha_{i1}.$$

In the case of spherical cavity filled with a highly viscous liquid, the tensor $\mathbf{P}$ introduced in [4] is diagonal with equal diagonal entries. For the sphere of radius a, we have

$$\tilde{\mathbf{P}} = P\,\mathrm{diag}(1,1,1), \quad P = \frac{8\pi\rho a^7}{525\vartheta}, \tag{1.6}$$

where the tensor $\tilde{\mathbf{P}}$ depends only on the cavity shape, ρ and ϑ are the density and the kinematic coefficient of viscosity of the liquid in the cavity, and a is the cavity radius.

The coefficients B_2^i and B_3^i in (1.5) have a similar form and are obtained by cyclic shift of the indices, the α_{ij} are the direction cosines between the reference frames Oy_i ($i = 1, 2, 3$) and Oz_i ($i = 1, 2, 3$), and p, q, r are the projections onto the axes Oz_i ($i = 1, 2, 3$) of the satellite angular velocity vector $\boldsymbol{\omega}$ with respect to the coordinate system $Ox_1x_2x_3$.

With the above assumptions about the smallness of the medium resistance and about the high viscosity of the liquid filling the cavity, the projections of the torque due to the viscous liquid forces in the cavity inside the body onto the axes Oy_i ($i = 1, 2, 3$) have the form

$$\begin{aligned} L_i^p = \frac{P}{A_1A_2A_3}\{&p[q^2A_2(A_1-A_2)(A_2-A_3+A_1) + r^2A_3(A_1-A_3)(A_3-A_2+A_1)]\alpha_{i1} \\ &+ q[r^2A_3(A_2-A_3)(A_3-A_1+A_2) + p^2A_1(A_1-A_2)(A_3-A_1-A_2)]\alpha_{i2} \\ &+ r[p^2A_1(A_3-A_1)(A_1-A_2+A_3) + q^2A_2(A_3-A_2)(A_2-A_1+A_3)]\alpha_{i3}\}, \quad i = 1, 2, 3, \end{aligned} \tag{1.7}$$

up to second-order infinitesimals.

In what follows, when studying the averaged system, it is convenient to use the kinetic energy T as an additional slow variable whose derivative has the form

$$\frac{dT}{dt} = \frac{2T}{G}L_3 + G\sin\theta\left[\cos\theta\left(\frac{\sin^2\varphi}{A_1} + \frac{\cos^2\varphi}{A_2} - \frac{1}{A_3}\right)(L_2\cos\psi - L_1\sin\psi)\right.$$

$$+\sin\varphi\cos\varphi\left(\frac{1}{A_1}-\frac{1}{A_2}\right)(L_1\cos\psi+L_2\sin\psi)\Big]. \tag{1.8}$$

We pose the problem of studying the satellite rotation evolution on a large time interval $t\sim\varepsilon^{-2}$, where the motion parameters vary significantly. We solve the problem by the averaging method [19].

2. PROCEDURE OF THE AVERAGING METHOD

Consider the unperturbed motion of system (1.1)–(1.8) (with $\varepsilon=0$) for the case in which the external torques are zero. In this case, the rotation of the rigid body is an Euler–Poinsot motion. The variables G, δ, λ, T, and ν become constant, and φ, ψ, and θ are functions of time t. In the perturbed motion, G, δ, λ, ν, and T are the slow variables, and the Euler angles φ, ψ, and θ are the fast variables.

Consider the rotation under the condition that $2TA_1\geq G^2>2TA_2$, which corresponds to the trajectories of the angular momentum vector surrounding the axis of the maximal moment of inertia A_1 [20]. We introduce the quantity

$$k^2=\frac{(A_2-A_3)(2TA_1-G^2)}{(A_1-A_2)(G^2-2TA_3)},\quad 0\leq k^2\leq 1, \tag{2.1}$$

which is a constant in the unperturbed motion, namely, the modulus of the elliptic functions describing this motion.

To construct the averaged system in the first approximation, we substitute the solution of the unperturbed Euler–Poinsot motion [20] into the right-hand sides of Eqs. (1.1) and (1.8) with regard to (1.3), (1.4), and (1.7), average with respect to the variable ψ, and then average with respect to time t, taking into account the dependence of φ and θ on t. The previous notation for the slow variables G, δ, λ, T, and ν is preserved. As a result, we obtain equations of the form

$$\begin{aligned}
\frac{dG}{dt}&=-\frac{G}{R(k)}\{I_{22}(A_1-A_3)W(k)+I_{33}(A_1-A_2)[k^2-W(k)]+I_{11}(A_2-A_3)[1-W(k)]\},\\
\frac{dT}{dt}&=-\frac{2T}{R(k)}\Bigg(I_{22}(A_1-A_3)W(k)+I_{33}(A_1-A_2)[k^2-W(k)]\\
&\quad+\frac{(A_1-A_2)(A_1-A_3)(A_2-A_3)}{S(k)}\left\{\frac{I_{33}}{A_3}[k^2-W(k)]+\frac{I_{22}}{A_2}(1-k^2)W(k)\right\}\\
&\quad+\frac{I_{11}}{A_1}\frac{(A_2-A_3)R(k)}{S(k)}[1-W(k)]\Bigg)-\frac{4PT^2(A_1-A_3)(A_1-A_2)(A_2-A_3)}{3A_1^2A_2^2A_3^2S^2(k)}\\
&\quad\times\Bigg(A_2(A_1-A_3)(A_1+A_3-A_2)\{(k^2-1)+(1+k^2)[1-W(k)]\}\\
&\quad+A_1(A_2-A_3)(A_3+A_2-A_1)[(k^2-2)W(k)+k^2]\\
&\quad+A_3(A_1-A_2)(A_1+A_2-A_3)[(1-2k^2)W(k)+k^2]\Bigg),\\
\frac{d\delta}{dt}&=-\frac{3\omega_0^2}{2G}\beta_2\beta_3N^*,\quad \frac{d\lambda}{dt}=\frac{3\omega_0^2}{2G\sin\delta}\beta_1\beta_3N^*.
\end{aligned} \tag{2.2}$$

Here we have introduced the notation

$$\begin{aligned}
N^*&=A_2+A_3-2A_1+3\left(\frac{3A_1T}{G^2}-1\right)\left[A_3+(A_2-A_3)\frac{K(k)-E(k)}{K(k)k^2}\right],\\
W(k)&=1-\frac{E(k)}{K(k)},\quad R(k)=A_1(A_2-A_3)+A_3(A_1-A_2)k^2,\\
S(k)&=A_2-A_3+(A_1-A_2)k^2.
\end{aligned} \tag{2.3}$$

Note that the expression in square brackets in the formula for N^* in (2.3) contains a removable singularity as $k\to 0$. Here $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kind, respectively.

It follows from Eqs. (2.2) and (2.3) that the medium resistance results in the evolution of both the angular momentum G and the body kinetic energy T, and the equations contain only the diagonal entries I_{ii} of the friction torque matrix. The terms containing the offdiagonal entries I_{ij} $(i \neq j)$ disappear after the averaging. The evolution of the body kinetic energy T is also affected by the torque due to the viscous liquid forces. The variations in the angles λ and δ depend on all forces acting on the body.

Consider the system composed of the equations for λ and δ in system (2.2), (2.3) and relation (1.2). We write them as

$$\dot{\delta} = \omega_0^2 \Delta(\nu, \delta, \lambda), \quad \dot{\lambda} = \omega_0^2 \Lambda(\nu, \delta, \lambda), \quad \nu = \omega_0 t, \tag{2.4}$$

where Δ and Λ are the coefficients on the right-hand sides in two equations in (2.2), δ and λ are slow variables, and ν is a semislow variable.

We obtain a system of special form, which we solve by the modified averaging method [21]. After averaging, we obtain

$$\dot{\delta} = 0, \quad \dot{\lambda} = \frac{3\omega_0^2 N^* \cos\delta}{4G}. \tag{2.5}$$

We preserve the notation for the slow averaged variables. Note that the action of the applied forces does not change the angular velocity δ and that the deviation of the vector $\mathbf{G}$ from the vertical remains constant in this approximation.

By differentiating the expression for k^2 (2.1) and by using equations (2.2) and (2.3) for the angular momentum and the kinetic energy, we obtain the differential equation

$$\frac{dk^2}{dt} = \frac{S(k)}{T(A_1 - A_2)(A_1 - A_3)(A_2 - A_3)} \left[R(k)\frac{dT}{dt} - S(k)G\frac{dG}{dt} \right]. \tag{2.6}$$

The system of equations consisting of the expressions (2.5), the first two equations in system (2.2), and equation (2.6) was integrated numerically. The integration was carried out for the initial conditions $k^2(0) = 0.99999$, $G(0) = 1$, $\delta(0) = 0.785$, and $\lambda(0) = 0.785$ and for the moments of inertia $A_1 = 8$, $A_2 = 5, 7$, and $A_3 = 4$. We considered two cases. In the first case, numerical calculations were performed for the drag coefficients $I_{11} = 2.322$, $I_{22} = 1.31$, and $I_{33} = 1.425$. The second numerical integration was performed for the drag coefficients $I_{11} = 2.6$, $I_{22} = 3.0$, and $I_{33} = 1.7$.

The initial value of the kinetic energy has the form

$$T_0 = \frac{G_0^2 s(k^2(0))}{2R(k^2(0))}.$$

The numerical results are illustrated in Figs. 1–5. Curves *1* and *2* in the figures correspond to the results obtained in the first case (for $A_2 = 5$ and $A_2 = 7$, respectively), and curve *3* corresponds to the second case.

Numerical analysis shows that the functions $T(t)$ and $G(t)$ are monotone decreasing (Figs. 1 and 2). The expression in curly brackets on the right-hand side in Eqs. (2.2) for T and G is positive (for $A_1 > A_2 > A_3$), because the inequalities $(1 - k^2)K \leq E \leq K$ are satisfied [22]. Therefore, $dT/dt < 0$, because $T > 0$; i.e., the variable T strictly decreases for any $k^2 \in [0, 1]$ (Fig. 1). The angular momentum G strictly decreases as well (Fig. 2).

Let us study the evolution of k^2 (Fig. 3). Since the function $T(t)$ tends to zero faster than the function $G(t)$, the expression in square brackets in (2.6) becomes

$$\left[R(k)\frac{dT}{dt} - S(k)G\frac{dG}{dt} \right] < 0.$$

The first two equations in system (2.2) and (2.6) were integrated numerically. Curves *1* and *2* correspond to the initial values of the parameters $I_{11} = 0.919$, $I_{22} = 5.228$, and $I_{33} = 1.425$.

Figure 4 presents the graph of variations in the angle λ of the angular momentum vector. The variations in the angle λ are caused by the gravitational attraction, drag forces, and the torque due to the viscous liquid forces in the body cavity. The character of variations in the angle λ is the same as in [13]. The increment $\lambda = \lambda(t)$ contains a nonconstant value N^* which depends on the functions $k^2 = k^2(t)$, complete elliptic integrals $K(k)$ and $E(k)$, and the kinetic energy and the angular momentum. The

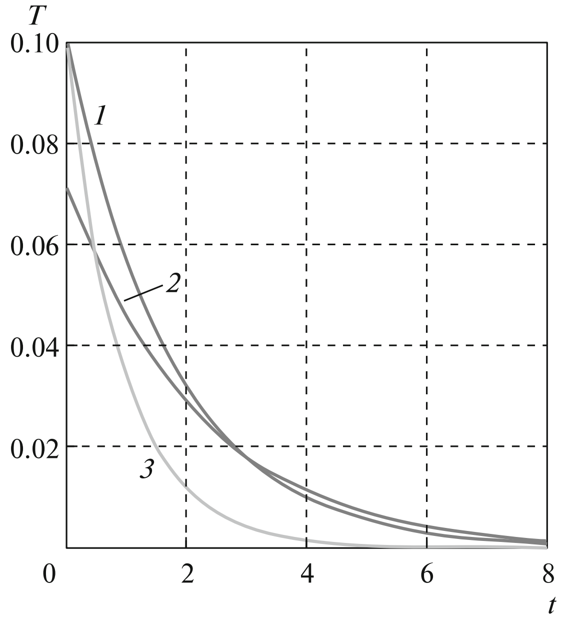

Fig. 1.

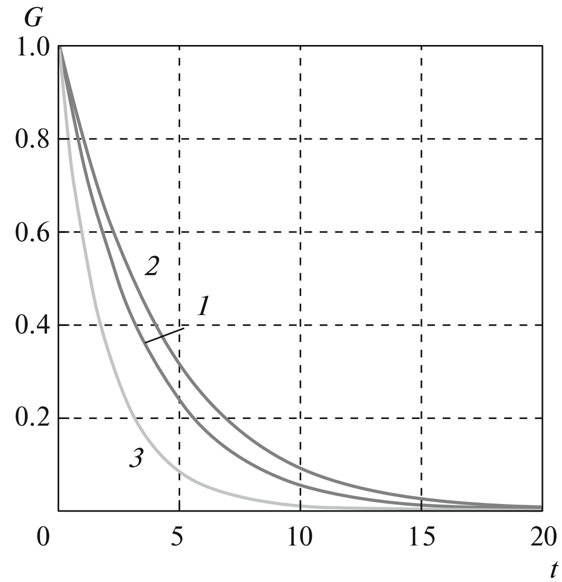

Fig. 2.

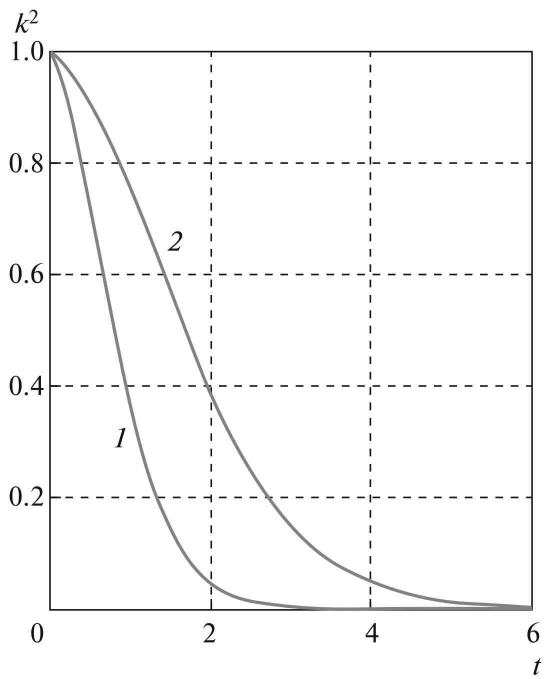

Fig. 3.

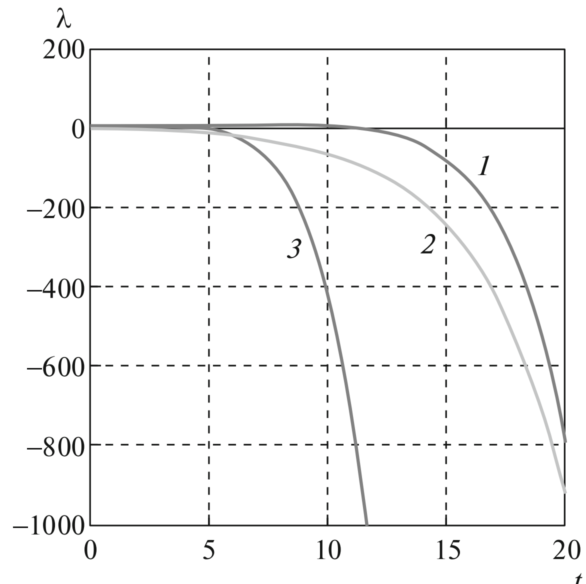

Fig. 4.

kinetic energy T and the angular momentum value G are decreasing functions, and so is $k^2 \to 0$. The value of N^* also decreases, but N^* is positive at the initial time, and hence the graph of variations in $\lambda(t)$ increases. The rate of decrease in the angle λ is proportional to T/G^3 (2.5), which tends to infinity with time. Therefore, λ decreases faster on a larger time interval. Curve *1* in Fig. 4 shows that the curvature of the function $\lambda(t)$ increases with decreasing moment of inertia A_2. A similar behavior is also exhibited as the drag coefficients increase (Fig. 4, curve *3*).

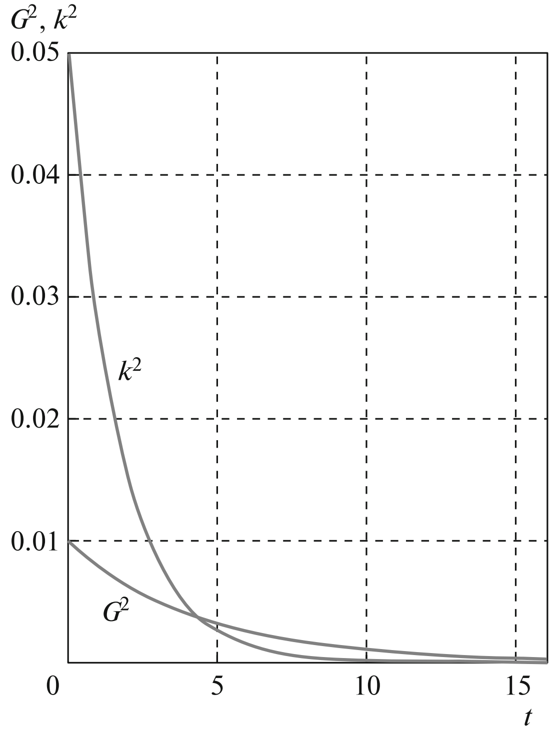

Fig. 5.

3. ANALYSIS OF THE LIMIT CASE

For small k^2, which corresponds to a nearly rotational motion about the axis A_1, the system of equations for G^2 and k^2 becomes

$$\frac{dG^2}{dt} = G(\chi + k^2\alpha), \quad \frac{dk^2}{dt} = k^2(\beta + G^2\gamma), \quad \chi = -\frac{I_{11}}{A_1}, \tag{3.1}$$

where

$$\alpha = -\frac{(A_1 - A_3)(I_{22}A_1 - I_{11}A_2) + (A_1 - A_2)(I_{33}A_1 - I_{11}A_3)}{2A_1^2(A_2 - A_3)},$$
$$\beta = -\frac{I_{33}A_1 - I_{22}A_3}{A_2A_3} - \frac{2(I_{22}A_1 - I_{11}A_2)}{A_1A_2}, \quad \gamma = -\frac{P}{A_1A_2A_3}\left(\frac{A_1 - A_2}{A_3} + \frac{A_1 - A_3}{A_2}\right).$$

We note that the resulting equations (3.1) formally coincide with the system of differential equations describing the evolution of ecological systems (the predator–prey system) [23, 24]. System (3.1) determines the first integral

$$k^\chi \exp(\alpha k^2) = C_1 G^\beta \exp(\gamma G^2), \quad C_1 = \text{const.} \tag{3.2}$$

For small G^2 and k^2, it follows from relations (3.1) and (3.2) that the character of variations in the squared angular momentum depends on the sign of the expressions $I_{22}A_1 - I_{11}A_2$ and $I_{33}A_1 - I_{11}A_3$, and the behavior of k^2 depends on the sign of the coefficient β. The coefficient γ is always negative. Numerical integration showed that for $\alpha < 0$ and $\beta < 0$ the body tends to the state of rest with increasing time (Fig. 5). For $\alpha > 0$ and $\beta > 0$, the functions $G^2(t)$ and $k^2(t)$ are periodic as in [23, 24].

Note that $G = 0$ and $k^2 = 0$ are stationary points of system (3.1). From Eqs. (3.1), we obtain $G^2 = G_0^2 \exp(\chi t)$ for $k^2(0) = 0$ and $k^2 = k_0^2 \exp(\beta t)$ for $G^2(0) = 0$. In this case, the function $G^2(t)$ decreases strictly, and the variations in the function $k^2(t)$ depend on the sign of β.

CONCLUSION

The motion of a dynamically asymmetric satellite with a spherical cavity filled with a strongly viscous liquid under the action of gravitational and medium resistance torques is studied. The resulting system of equations of motion contains fast and slow variables. The method of averaging over the Euler–Poinsot motion and the modified averaging method were applied successively. The variations in the medium angular momentum depend only on the medium resistance, and the variations in the kinetic energy also depend on the influence of the cavity filled with the liquid. The evolution of the orientation angle λ of the angular momentum vector depends on the gravitational attraction, the drag forces, and the torque due to the viscous liquid forces in the body cavity. In the second approximation of the averaging method, the angle of the angular momentum deviation from the vertical remains constant. The values of the angular momentum and the kinetic energy decrease monotonically. The influence of the viscous liquid forces is small compared with the action of the gravitational and drag torques.

ACKNOWLEDGMENTS

The work was partially supported by the Third Joint Competition of the Ukrainian and Russian Foundations for Basic Research (grants nos. 953.1/010 and 13-01-90411 Ukr_f_a), by the Russian Foundation for Basic Research (grant no. 13-01-00180), and by the Program NSh-2710.2014.1.

REFERENCES

1. V. V. Beletskii, *Artificial Satellite Motion about Its Center of Mass* (Nauka, Moscow, 1965) [in Russian].
2. F. L. Chernous'ko, "On the Motion of a Satellite about Its Center of Mass under the Action of Gravitational Torques," Prikl. Mat. Mekh. **27** (3), 474–483 (1963) [J. Appl. Math. Mech. (Engl. Transl.) **27** (3), 708–722 (1963)].
3. V. V. Beletskii, *Satellite Motion about the Center of Mass in Gravitational Field* (Izd-vo MGU, Moscow, 1975) [in Russian].
4. F. L. Chernous'ko, "Motion of a Rigid Body with Cavities Filled with Viscous Fluid at Small Reynolds Numbers," Zh. Vychisl. Mat. Mat. Fiz. **5** (6), 1049–1070 (1965) [U.S.S.R. Comput. Math. Math. Phys. (Engl. Transl.) **5** (6), 99–127 (1965)].
5. V. N. Koshlyakov, *Problems of Solid Mechanics and Applied Theory of Gyros: Analytic Method* (Nauka, Moscow, 1985) [in Russian].
6. L. D. Akulenko, D. D. Leshchenko, and F. L. Chernous'ko, "Fast Motion of a Heavy Rigit Body about a Fixed Point in a Resistive Medium," Izv. Akad. Nauk SSSR. Mekh. Tverd. Tela, No. 3, 5–13 (1982) [Mech. Solids (Engl. Transl.) **17** (3), 1–8 (1982)].
7. M. Iñarrea and V. Lanchares, "Chaotic Pitch Motion of an Asymmetric Nonrigid Spacecraft with Viscous Drag in a Circular Orbit," Int. J. Non-Lin. Mech. **41**, 86–100 (2006).
8. L. D. Akulenko, D. D. Leshchenko, and A. L. Rachinskaya, "Evolution of the Satellite Fast Rotation Due to the Gravitational Torque in a Resisting Medium," Izv. Akad. Nauk. Mekh. Tverd. Tela, No. 2, 13–26 (2008) [Mech. Solids (Engl. Transl.) **43** (2), 173–184 (2008)].
9. E. P. Smirnova, "Stabilization of Free Rotation of an Asymmetric Top with Cavities Completely Filled with a Fluid," Prikl. Mat. Mekh. **38** (6), 980–985 (1974) [J. Appl. Math. Mech. (Engl. Transl.) **38** (6), 931–935 (1974)].
10. E. P. Osipov and R. S. Sulikashvili, "On Oscillations of a Rigid Body with a Cavity Completely Filled with a Viscous Liquid in an Ellipsoidal Orbit," Trudy Tbilis. Matem. Inst. Akad. Nauk GSSR **58**, 175–186 (1978).
11. L. D. Akulenko and D. D. Leshchenko, "Rapid Rotation of a Heavy Gyrostat about a Fixed Point in a Resisting Medium," Prikl. Mekh. **18** (7), 102–107 (1982) [Int. Appl. Mech. (Engl. Transl.) **18** (7), 660–665 (1982)].
12. V. V. Sidorenko, "Evolution of the Rotational Motion of a Planet with a Liquid Core," Astron. Vestnik **27** (2), 119–127 (1993) [Solar Syst. Res. (Engl. Transl.) **27** (2), 201–208 (1993)].
13. L. D. Akulenko, D. D. Leshchenko, and A. L. Rachinskaya, "Evolution of Rotations of a Satellite with a Cavity Filled with a Viscous Liquid," Mekh. Tverd. Tela, No. 37, 126–139 (2007).
14. D. D. Leshchenko and S. G. Suksova, "On the Motion of an Asymmetric Gyro in a Resisting Medium,' J. Intern. Federation of Nonlinear Analysts – Acad. Nonlinear Sci. "Problemy Nelineinogo Analiza v Inzhenernykh Sistemakh" **9** (2)(18), 83–89 (2003).
15. E. Yu. Baranova and V. G. Vilke, "Evolution of Motion of a Rigid Body with a Fixed Point and an Ellipsoidal Cavity Filled with a Viscous Fluid," Vestnik Moskov. Univ. Ser. I Mat. Mekh., No. 1, 44–50 (2013) [Moscow Univ. Mech. Bull. (Engl. Transl.) **68** (1), 15–20 (2013)].

16. L. D. Akulenko, Ya. S. Zinkevich, D. D. Leshchenko, and A. L. Rachinskaya, “Rapid Rotations of a Satellite with a Cavity Filled with a Viscous Fluid under the Action of Moments of Gravity and Light Pressure Forces,” Kosmich. Issled. **49** (5), 453–463 (2011) [Cosmic Res. (Engl. Transl.) **49** (5), 440–451 (2011)].
17. D. D. Leshchenko, A. L. Rachinskaya, and Yu. S. Shchetinina, “Evolution of Rotations of a Symmetric Gyro in a Gravity Field and in a Resisting Medium,” Makhanika Tverd. Tela, No. 42, 93–102 (2012).
18. L. D. Akulenko, D. D. Leshchenko, A. L. Rachinskaya, and Ya. S. Zinkevich, *Perturbed and Controlled Rotations of a Rigid Body* (Mechnikov Odessa Nation. Univ., Odessa, 2013) [in Russian].
19. V. M. Volosov and B. I. Morgunov, *Averaging Method in the Theory of Nonlinear Oscillatory Systems* (Izdat. MGU, Moscow, 1971) [in Russian].
20. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 1: *Mechanics* (Nauka, Moscow, 1973; Pergamon Press, Oxford, 1976).
21. L. D. Akulenko, “Higher-Order Averaging Schemes in Systems with Fast and Slow Phases,” Prikl. Mat. Mekh. **66** (2), 165–176 (2002) [J. Appl. Math. Mech. (Engl. Transl.) **66** (2), 153–163 (2002)].
22. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Sums, Series, and Products* (Nauka, Moscow, 1971) [in Russian].
23. E. Kamke, *Reference Book in Ordinary Differential Equations* (Van Nostrand, New York, 1960; Nauka, Moscow, 1971).
24. V. Volterra, *Mathematical Theory of the Struggle for Existence* (Nauka, Moscow, 1976) [in Russian].