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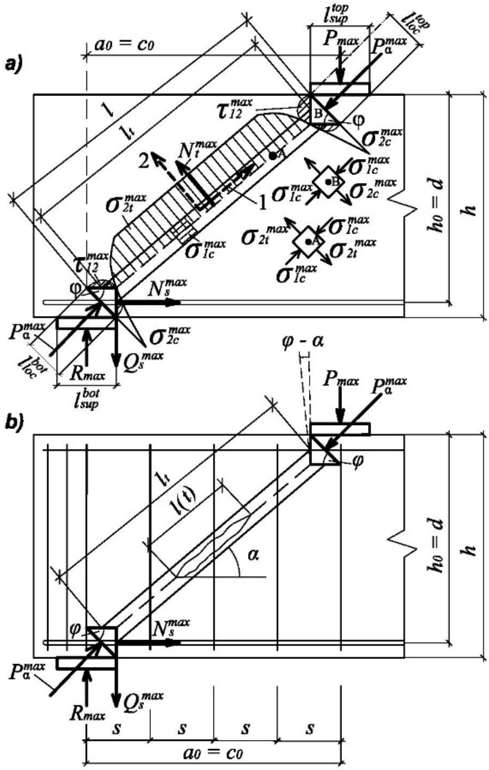
Basis of force and deformation-force resistance of reinforced concrete at the complex stress-strain state



UNIVERSITY NORTH, Koprivnica/Varaždin

and

ODESSA STATE ACADEMY OF CIVIL ENGINEERING AND ARCHITECTURE

BASIS OF FORCE AND DEFORMATION-FORCE RESISTANCE OF REINFORCED CONCRETE AT THE COMPLEX STRESS-STRAIN STATE

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CONTENT

INTRODUCTION

World practice shows that the volumes of using monolith and prefabricated reinforced concrete are increasing, and in the coming decades they will remain the main structural material. Therefore, it is obvious that the effective using of reinforced concrete, is possible with the presence of reliable, accurate and economical calculation methods. Thereby, much attention is paid towards the development of the theory of reinforced concrete.

The experience of designing, construction and exploitation of span reinforced concrete structures shows that, practically, all of them work at complex stress-strain state. At the same time, the researchers pay much more attention to the calculation of strength, deformability and crack-resistance of elements in normal sections, than to the calculation of their areas near supports, including the inclined sections.

Besides, a significant number of span reinforced concrete elements, is subjected to the action of low-cycle loads. They lead to such specific features of concrete work as nonlinearity of deformation, micro cracking, accumulation of residual deformations, low-cycle fatigue (fatigue damage), decompaction of concrete etc. The results of the research show that the destruction of structure under the action of low-cycle loads occurs at the smaller stresses, than the destruction of structure under the action of short duration static load.

CHAPTER 1 FUNDAMENTALS OF FORCE AND FORCE-DEFORMATION RESISTANCE OF REINFORCED CONCRETE IN TERMS OF ITS COMPLEX STRESS-STRAIN STATE

The data given in this chapter of monograph are based on the research of Romashko V.M. [3].

Analysis of modern studies of concrete and reinforced concrete structures was performed. It has shown that the overall theory of concrete and reinforced concrete continues to develop in the direction of compliance with generally accepted principles and preconditions of solids mechanics. Even though the problem of the general theory of resistance of concrete and reinforced concrete creation is far from perfect, certain ways to solve it have already been developed.

According to numerous studies, summarized in Bondarenko V.M. work [1], following studies hierarchy is considered to be methodologically correct: physical model, calculation model, and mathematical model.

The physical model should include the fullest possible description of the object of research in the physical objective terms. Obviously, the physical model cannot be created by purely empirical observation, since it is impossible to understand the experiment without analytical thinking and experimental data synthesis. Physical model creation is based on a synthesis of information storage array, sometimes chaotic, and contradictions, empirical and intuitive nature of related industries data and analogies with the following formulation of the initial principles and regulations, sometimes contradictory to traditional notions. Physical model should include, without any simplifications, all known functions and other relations and links between process parameters, which can be, both, deterministic and stochastic.

However, the lack of certainty, and sometimes an excessive complexity of the relations between the factors and difficulties of logical and mathematical interpretations, necessitate the transition to the next stage of research- to the calculation model.

The computational model, being released from minor and insignificant factors, replacing or complementing the primary information by means of hypotheses and invariants, thus simplifying the physical model makes it, firstly, an engineering graceful, and secondly solving in a modern way. However, the transition from the physical model to the calculation, for example, by means of linearization and averaging the time processes, needs to be done very carefully to keep the right decision, not to aggravate the quality of the processes, to ensure sufficient accuracy of the results. Excessive simplification often does more harm than good.

Computational models, as a rule, allow to determine the form and structure of the solutions that are expected. It is necessary to ensure the selected models adequacy.

At the same time, the implementation of the computational model, the engineering attainability and significance of the results, depend on the applied mathematics. The mathematical model is a set of equations, and other relations, algorithms and solutions, as well as the programs agreed with the possibilities of the existing computer equipment. Mathematical model has to be reproduced.

However, it should be noted that not all computational and mathematical models can be given in the form adapted for the development of engineering calculation methods. So, only full and unambiguous enough representations or research processes (physical model) and a reasoned setting of specific initial hypotheses and invariants (calculation model), statistical evaluation of experiment data, development of the empirical approximations do not contradict the general concepts, as well as successful mathematical implementation (mathematical model) may result in a theoretical study of the problem with reliable results.

Thus, solids mechanics, as a part of continuum mechanics, is the basis for the theory of the structures force resistance, in its particular sections as structural mechanics, strength of materials, theory of elasticity, plasticity, creep, and fracture mechanics of materials, rock and soils mechanics, design of structures, foundations and cellars.

A distinctive feature of the reinforced concrete force resistance, in addition to the anisotropy and irreversibility, is the mode-genetic specificity of nonlinear, unbalanced deformation. Ignoring this fact (for example, an existing spring-piston simulation), undoubtedly, leads to qualitative losses and quantitative errors. Therefore, modern science and computational engineering studies are developing in a phenomenological way, according to the fundamental provisions of mechanics, physics and thermodynamics. The logical framework of the phenomenological method, as it is known, consist of research and statistical evaluation of deformation and fracture factors of materials and structures, the identification and analysis of the existing qualitative and quantitative relationships between them, a generalization of the results obtained with the formulation system of hypotheses and invariants, enough to create an application theory of solving engineering problems, including the definition of elastic and inelastic deformations, creep deformation and others.

Beneath strain-force model of reinforced concrete structures, resistance force and other effects, mean a prototype of a real process of deformation of concrete and reinforced concrete elements and structures, reproduced by some of the generalized state diagram.

Deformation-force model is also discrete. In cross section of the element or structure it is reproduced with a large number of elementary areas, within which the properties of the material are taken as constant. The continuity of the curvature function is provided on the edge of cracking by the same generalized state diagram of reinforced concrete elements and structures, in the form of continual dependence without breaks, fractures and jumps.

In addition, the entire process of deformation of concrete and reinforced concrete elements and their actual stress-strain diagrams, are reproduced in the strain-force models using the stiffness function. This connects the main force and deformation parameters of the stress-strain state of these elements and structures. Thus, as shown by experiments, up to the loss of elements' bearing capacity, stiffness nonlinearly depends on these factors. Application of the extreme criterion [2], which fixes the moment of the reinforced concrete element bearing capacity loss, and the force limits of deformation parameters, only emphasizes the power orientation of the model. Deformation orientation of the model is realized via deformation in the equations of equilibrium and distribution law in the section height (usually flat sections hypothesis).

It should be emphasized that the basis of modern deformation models of reinforced concrete elements and structures resistant to external action lies in discretization of design schemes and their representation in the form of a elements set of certain structural levels. In particular, in work [3] a classification of the objects of study in five interconnected levels of hierarchical subordination are proposed to be introduced:

- material
- section (a set or a combination of materials)
- element (a set or a combination of sections)
- construction (a set or a combination of elements)
- structural scheme of the building or structure (a set or a combination of elements and constructions)

1.1 Classical approaches to determining the stress-strain state of reinforced concrete structures

The theory of resistance of reinforced concrete structures, as mentioned before [4], is aimed at the precise definition of the four major problems:

- accurate calculation of the load at which the first cracks appear;
- determining the width of the cracks in the operational phase, starting from the moment of their appearance;
- calculation of rigidity and deflections, including the maximum permissible;
- definition of maximum possible bearing capacity (strength or stability).

Thus, under the influence of external loads or impacts, the internal forces in the most intense section of reinforced concrete elements, as well as deformation of concrete in its extreme fibre, increase from zero to some limits. With double-digit stress diagram, the section usually goes through three typical stages of stress-strain state.

Stage 1 (operation without fractures) is observed at relative elongation concrete tension area smaller than the limit value of ε_{ctu}. At this stage, the deflection value and the rigidity of low reinforced structures operating without cracks in the tension zone, are calculated.

Stage 1a occurs at the moment when the relative elongation deformation of concrete extreme fibres reaches the limit values ε_{ctu}. Concrete starts to burst on and off from work, causing the reinforcement to work hard. An accurate assessment of stress-strain state of the cross sections at this stage will allow to determine the force at which the first cracks appear in the concrete tension area and calculate the element's section stiffness (without cracks).

Stage 2 occurs after the appearance of the first cracks in the concrete tension area. It is considered as the main working stage of deformation of reinforced concrete bent elements. At this stage researches can determine the width of the crack opening, and calculate the stiffness and the magnitude of the deflection of reinforced concrete elements under the action of operating loads.

Stage 2a represents the limit state. It occurs when the stress in the tension reinforcement reaches the characteristic strength values on the verge of its yield f_{yk} or extreme fibre stress of compressed concrete − characteristic values of the compressive strength of f_{ck}, i.e, reinforced concrete element begins to form so-called "plastic hinge" in the cross section (start of the destruction).

The final stage 3 represents the limit stress-strain state of reinforced concrete elements and characterizes its complete destruction. It occurs when the balance of forces in the most stressed section element can no longer be ensured. At the same time, the relative deformation of the extreme fibres of compressed concrete reach the value $\varepsilon_{cu} > \varepsilon_{c1}$ and tension reinforcement can work, both, before and on the border of yield:

$$\varepsilon_s < \varepsilon_{s0},\ \varepsilon_{s0} \leq \varepsilon_s \leq \varepsilon_{suk}.$$

Researches constructions on stages 2a and 3 make it possible to determine the real value of the destructive forces that should be displayed in the standards.

As known from mechanics of solid deformable body two main and four additional conditions are needed for the task. There are two equilibrium equations for planar systems in a normal section, as well as auxiliary terms in the form of strain distribution law along the section height of the element $1/r = (\varepsilon_c + \varepsilon_s) / d$, physical dependence between stresses in the reinforcement $\sigma_s = f(\varepsilon_c)$, and the compressed and tensioned zones of the concrete $\sigma_c = f(\varepsilon_c)$ i $\sigma_{ct} = f(\varepsilon_{ct})$.

It should be noted that the mapping of stages of the stress-strain state of reinforced concrete structures for force model of the previous standards [5] has a number of serious shortcomings:

- simplified method of accounting for concrete plastic deformations in the form of a rectangular stress distribution;
- adopted in [5] is approach of strength calculation of reinforced concrete elements that cannot be directly implemented in the calculations of their strenght and resistance to fracture;

- calculation method of the work impact of tensioned concrete between and above the cracks on the overall stress-strain state of reinforced concrete element weakly reflects the redistribution of forces between the concrete in tension zone and tensioned reinforcement;
- a characteristic of strength tensile reinforcement on the verge of its yield f_{yk}, or the characteristic value of concrete compressive strength of f_{ck} in its extreme fibroids, cannot act as the exhaustion criteria of bearing capacity of reinforced concrete structures.

Deformation model, as opposed to force, more accurately reflects the stress-strain state of reinforced concrete elements in the limit stage. However, a single general criterion of the exhaustion of bearing capacity has not yet been developed in these models.

If we assume that the real model of deformation of concrete and reinforced concrete structures is [3] the deformation force in nature, in its framework, the only common criterion of the exhaustion of bearing capacity occurs in moment of imbalance of strenght that is secured by an extreme criterion of bearing capacity $dM / d(1/r)$.

So, the real state of the reinforced concrete structure cannot be displayed only by stress distribution or deformations diagrams. This can only be done when both diagrams are used in conjunction. In this case, the generalized model of the element deformation should be able to equally reflect both the nature of the growth of relative deformation of materials, and the process of continuous redistribution of stresses in them, especially at stages that are close to the limit equilibrium.

Loss of reinforced concrete elements of the bearing capacity of normal cross sections is characterized by a violation of one of the two known equilibrium equations $\Sigma N = 0$ and $\Sigma M = 0$. More rigid is the second equation, which implies defining the condition of limit equilibrium:

$$M_{Ed} \leq M_u \tag{1.1}$$

where M_{Ed} − estimated value of the bending moment on the external load;

M_u − limit value of the moment of internal forces in the cross section of reinforced concrete element.

In the design of concrete and reinforced concrete structures maximum bearing capacity is M_u or N_u by $dM / d(1/r) = 0$ or $dN/d\varepsilon = 0$ according to [3].

The characteristic that links the strength (*M*, *N*) and deformational (*1/r*, *ε*) parameters can be stiffness of the element in a certain section. The knowledge patterns of change in the rigidity are critical, not only in the calculation of reinforced concrete structures for deflections and crack resistance, but also in determining its load-bearing capacity.

It is known that the rigidity of the concrete or reinforced concrete element is an integral characteristic. Obviously, when the axial compression or tension of concrete element with small or occasional eccentricities varies, primarily or only by changing the concrete strain module (fig. 1.1, a), since all the geometric parameters of such elements remain unchanged:

$$D_{cc}=E_{cc}^{\text{int}}\cdot I_{cc}\ ,\ \ D_{ct}\ =\ E_{ct}^{\text{int}}\ \cdot I_{ct} \qquad (1.2)$$

where $E_{cc(t)}^{\text{int}}$ – integrated (averaged) module of compressed or stretched concrete strain;

$I_{cc\ (t)}$ – moment of inertia of the compressed or tensioned concrete section.

With double stress diagram in the reinforced concrete elements cracks can appear and develop, break tensile strength of reinforcement with concrete and its gradual removal from service. The value of the integral rigidity of these elements consists of stiffness of compressed and stretched zones of concrete, compressed and stretched reinforcement:

$$D = E_{cc}^{\text{int}} I_{cc} + E_{cc}^{\text{int}} I_{ct} + \sum E_{sc} I_{sc} + \sum E_{st} I_{st} \qquad (1.3)$$

The function of rigidity in the section with a crack (Fig. 1.1, b) should reflect the relatively rapid (almost sudden) exclusion from the work of the concrete in tension and the corresponding redistribution of the stresses in the tension zone, from the concrete to the reinforcement.

In the block between the cracks a reinforced concrete element stiffness varies by not geometric and deformation characteristics (modulus of concrete strain) but also can be reflected by the expression (1.3).

The stress-strain state of a reinforced concrete elements at this time should describe the characteristics of a calculated (averaged) section of the block between the cracks, including the averaged integral rigidity, which will change as the expression (1.3), relatively smoothly, with no apparent jumps of function.

Calculation of the average integral stiffening of the reinforced concrete structure by the expression (1.3) is a difficult task with many unknowns associated, not only with the deformation characteristics of the materials, but also with geometric section parameters. Therefore, in practical calculations average integral stiffness is suitably in its classical expression:

$$D = M / (1/r) \tag{1.4}$$

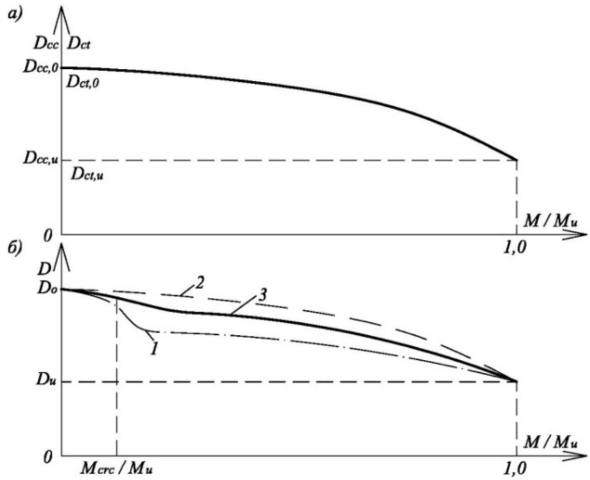

Fig 1.1. Modifying stiffness of the compressed (tensioned) concrete element with a small eccentricity

(a) and the diagram of the integral (averaged) bending stiffness of a reinforced concrete element

(b) in section: 1 – with crack; 2 – between cracks; 3 – averaged in block between cracks.

Obviously, connection between the stiffness and emerging internal forces in the section element and its curvature has to be nonlinear, capable of reproducing its stress-strain state only with simultaneous use of force and deformation characteristics.

In real conditions, even if the axial load concrete and reinforced concrete elements stiffness is changing, both, due to the deformation properties of materials (E), and due to the geometric parameters of cross-section (I), that is, $M / (1/r) = EI$.

In general, the stiffening of the element is associated, not only, with the level of efforts exposure M / M_u, but also with its deformation level $(1/r)$ /

($1/r_u$). Therefore, the force that an element has to take over, by work [3] can be calculated depending on its level of deformation by the formula:

$$M = \frac{E_{c0} I_{red,0} \cdot 1/r - M_u \left(\frac{1/r}{1/r_u} \right)^2}{1 + \left(\frac{E_{c0} I_{red,0}}{M_u} - \frac{2}{1/r_u} \right) \frac{1}{r}} \tag{1.5}$$

where M_u – bearing capacity of the concrete or reinforced concrete rod;

$1/r_u$ – element curvature in limit state;

$E_{c0} I_{red,0}$ – initial reduced stiffness of the concrete or reinforced concrete element section.

It is obvious that the expression (1.5) describes the state diagram of the element or structure (Fig. 1.2) with the ascending and descending deformation branches. In this case, the descending branch of the diagram is not possible to statically determine reinforced concrete structures and concrete elements because achievement of the ultimate state in the most intense section will lead to geometric variability and destruction.

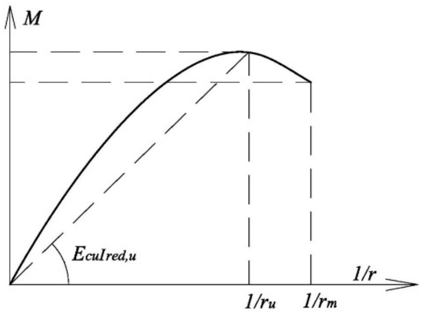

Fig 1.2. Relation between the curvature and the moment in the reinforced concrete structure (state diagram)

Structural analysis of the expression (1.5) shows that it describes a smooth and monotonic function of the ascending and descending branches of the diagram, and can, therefore, be unconditionally used for reinforced concrete structures, operating without formation of the cracks in the tension zone. With the emergence of cracks, their stiffness will change more rapidly due to the accelerated elimination of the work of concrete in tension and increasing of the curvature *1/r*.

Obviously, the rapid drop in stiffness (increasing of the curvature) occurs as a result of intense changes in the geometric characteristics of its cross-section. The majority of foreign scientists associates the stiffness of reinforced concrete structures with the, so-called, effective moment of inertia, which depends on the level of loading. Typically, it is done by adjusting the effective moment of inertia I_e using different degree dependences [6−13].

Likewise, it is advisable to adjust the curvature itself and the reinforced concrete structures with cracks in the tension zone, using functions like [3]:

$$\psi_i = 1 + m \cdot M / M_u (1 - M / M_u) \qquad (1.6)$$

where m − option, which is recommended [3] to be taken as depending on character of the destruction:

m = 3 at the work reinforcement yield in low reinforced elements ($\rho_l \leq$ 1,5%);

m = 2 at the yield in normally reinforced elements (1,5% < ρ_l < 3,5%);

m = 1,5 at concrete smashing in over reinforced elements (ρ_l > 3,5%) and in structures with high-strength reinforcement.

If in the equation (1.5) the replacement of $1/r$ with the adjusted (real) curvature $1/r^*$ of the reinforced concrete element with cracks by $1/r = (1/r^*)\,\psi_i$, can be carried out, the traditional entry of the phase diagram can be saved in practical calculations in the form of:

$$M = \frac{E_{c0} I_{red,0} \cdot \dfrac{1/r^*}{\psi_i} - M_u \left(\dfrac{1/r^*}{\psi_i \cdot 1/r_u} \right)^2}{1 + \left(\dfrac{E_{c0} I_{red,0}}{M_u} - \dfrac{2}{1/r_u} \right) \cdot \dfrac{1/r^*}{\psi_i}} \qquad (1.7)$$

In the work [3] it is shown, that seeking a universal function $\sigma_c = f(\varepsilon_c)$ based on the results of experimental studies of standard samples, is impossible, because in nature there are no two equivalent elements of the reinforced concrete, for which the stress-strain state would be the same. Therefore, a universal function should be required, not for the concrete stress-strain diagram and to the phase diagram of the RC element M − ($1/r$), which can be represented by irregular fractional-rational functions (1.5) or (1.7), since they allow us to describe the stress-strain state of a bent, and centrally or eccentrically compressed, or tensile RC element. Thus, it is evident that the material deformation diagram is a state diagram of standard sample, under the standard conditions of testing. If applied to the generalized state diagram of reinforced concrete element $M - (1/r)$, limit equilibrium hypothesis, and extreme criterion of the bearing capacity $dM / d(1/r) = 0$, you can get the actual diagram of the deformation of the compressed or tensile concrete.

So, the real model of concrete and reinforced concrete elements and structures, is always a deformation-force and cannot be solely deformation or solely force.

1.2 Strength calculation of normal sections of concrete and reinforced concrete elements of the first limit states group

Following hypotheses and conditions are laid into the basis of calculation of the strength of normal sections of these elements when they are arbitrary bent, compressed or tensed:

1. Discusses the elements and structures in which the lateral force influence on deflections is small and in the cross sections of which there is no torsion deformation.
2. The actual stress-strain state of these elements is described by the corresponding diagram in the form of improper fractional-rational function (1.5) or (1.7).
3. The process of deformation of reinforced concrete elements for their average sections is considered as equitable as the hypothesis of flat sections (fig. 1.3): $1/r = \varepsilon_c / x = \varepsilon_s / x_s = (\varepsilon_c - \varepsilon_s) / d = (\varepsilon_c - \varepsilon_s) / (x + x_s)$.
4. Load bearing capacity (strength) of concrete or of reinforced concrete element and the resistance of its design section to the load is considered to be exhausted in violation of classical conditions of the limit equilibrium $N \le N_u$ and $M \le M_u$.

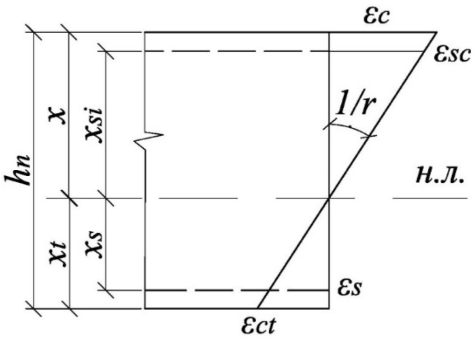

Fig 1.3. Character of the deformation changes in the averaged cross section of reinforced concrete element

5. The initial modulus of the deformation (initial modulus of elasticity) of concrete is calculated by the formula

$$E_{c0} = k_0 \cdot E_{cm} \tag{1.8}$$

with use of table data E_{c0} and k_0.

6. The connection between stress and strain of compressed and tensed concrete is described by its complete deformation diagrams in the form of improper rational functions:

$$\sigma_c = \frac{a \cdot \varepsilon_c - b_1 \cdot \varepsilon_c^2 / \varepsilon_{c1}}{1 + c_1 \cdot \varepsilon_c / f_{ck}} = \frac{a \cdot \varepsilon_c - b \cdot \varepsilon_c^2}{1 + c \cdot \varepsilon_c} \tag{1.9}$$

where

$$a = E_{c0};\ \ b = b_1 / \varepsilon_{c1} = f_{ck} / \varepsilon_{c1}^2;\ \ c = c_1 / f_{ck} = E_{c0} / f_{ck} - 2 / \varepsilon_{c1} \tag{1.10}$$

Taking into account (1.10), the expression (1.9) is transformed into the formula:

$$\sigma_c = \frac{E_{c0} \cdot \varepsilon_c - f_{ck} \cdot \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2}{1 + \left(\frac{E_{c0}}{f_{ck}} - \frac{2}{\varepsilon_{c1}}\right) \cdot \varepsilon_c} \tag{1.11}$$

$$\sigma_{ct} = \frac{E_{c0} \cdot \varepsilon_{ct} - f_{ctk} \cdot \left(\frac{\varepsilon_{ct}}{\varepsilon_{ct1}}\right)^2}{1 + \left(\frac{E_{c0}}{f_{ctk}} - \frac{2}{\varepsilon_{ct1}}\right) \cdot \varepsilon_{ct}} \tag{1.12}$$

7. Critical deformations of compressed and tensed concrete at the maximum compressive and tensile loads are determined by the respective formulas:

$$\varepsilon_{c1} = k_{el} \cdot \varepsilon_{c1,\,el} + k_{pl} \cdot \varepsilon_{c1,\,pl} \tag{1.13}$$

$$\varepsilon_{ct\,1} = k_{t,\,el} \cdot \varepsilon_{ct\,1,\,el} + k_{t,\,pl} \cdot \varepsilon_{ct\,1,\,pl} \tag{1.14}$$

where k_{el} ($k_{t,\,el}$) and k_{pl} ($k_{t,\,pl}$) − factors, taking into account the peculiarities of the concrete, work in the section element and the development of its elastic and plastic deformations. In "hard" mode, loading ($d\varepsilon_c / dt = const = d\varepsilon_{ct} / dt$) $k_{el} = k_{t,\,el} = k_{pl} = k_{t,\,pl} = 1$ based on (1.13), (1.14) are transformed into expressions:

$$\varepsilon_{c1} = \frac{f_{ck}}{E_{c0}} + (140 - 0{,}7 \cdot f_{ck}) \cdot 10^{-5} \tag{1.15}$$

$$\varepsilon_{ct\,1} = f_{ctk} \;/\; E_{co} + (3 - f_{ctk} \;/\; 3) \cdot 10^{-5} \tag{1.16}$$

Under standard conditions test with a "soft" load mode ($d\sigma_c / dt = const = d\sigma_{ct} / dt$), the critical strain ε_{c1} and $\varepsilon_{ct\,1}$ is recommended [3] to be described by equations (1.15) and (1.16), taking $k_{pl} = k_{t,\,pl} = 0{,}78$, and $k_{el} = k_{t,\,el} = 1{,}05$ for concretes with strength $f_{ck} \le 40$ N/mm^2 and $f_{ctk} \le 2{,}1$ N/mm^2;
$k_{el} = k_{t,\,el} = 1{,}03$ for concretes with strength 40 $N/mm^2 < f_{ck} \le 80$ N/mm^2 and 2,1 $N/mm^2 < f_{ctk} \le 3{,}3$ N/mm^2;
$k_{el} = k_{t,\,el} = 1{,}0$ at $f_{ck} > 80$ N/mm^2 and $f_{ctk} > 3{,}3$ N/mm^2.

8. Limit strains of boundary fibres of compressed concrete ε_{cu} are dependent on the stress-strain state of the element with a rectangular cross-section and the parameters of its reinforcement so it is recommended to determine them by using the general dependence:

$$\eta_{\varepsilon u} = \frac{\varepsilon_{cu}}{\varepsilon_{c1}} = 1 + 5^3 \cdot \varepsilon_s \cdot \frac{(k-1)}{(6-k)} \cdot \sum_{i=1}^{n} \rho_{li} \cdot \left(\frac{x_{si}}{x}\right)^2 + \beta_F \times$$
$$\times \left[\frac{\left(0{,}43 - 0{,}2(0{,}4 - m_h)^2\right)\left(1 - m_h^{3/2}\right)\left(1 - m_h / k\right)\sqrt{(\ln k)^{\left(1 - m_k^{3/2}\right)}}}{1 + \left(\left(l/6 - 0{,}1 m_h^2\right)(k-2)\ln\left(6/k - 2(0{,}1 - m_h)\right)\right)^2} + \right.$$
$$\left. + \frac{\left(0{,}43 - 0{,}2(0{,}4 - m_b)^2\right)\left(1 - m_b^{3/2}\right)\left(1 - m_b / k\right)\sqrt{(\ln k)^{\left(1 - m_b^{3/2}\right)}}}{1 + \left(\left(l/6 - 0{,}1 m_b^2\right)(k-2)\ln\left(6/k - 2(0{,}1 - m_b)\right)\right)^2} \right] \tag{1.17}$$

where m_h − parameter of inhomogeneous deformation along the section height h: $m_h = \varepsilon_{c0,h} / \varepsilon_{c2}$; $m_b = \varepsilon_{c0,b} / \varepsilon_{c2}$; $\varepsilon_{c2} = \varepsilon_{cu}$, a $\varepsilon_{c0} = 0$;

$\eta_{\varepsilon u}$ – the level of ultimate strains in the most compressed fibre of the concrete

$$\eta_{\varepsilon u} = \varepsilon_{cu} / \varepsilon_{c1}$$

x_{si} – the distance from the neutral line to the centre of gravity of compressed rods at $\sigma_{si} < f_{yk}$. If $\sigma_{si} \geq f_{yk}$, $x_{si} = 0$.

$\rho_{li} = A_{si} / (b_n h_n)$ – ratio of section reinforcement with the same rods;

β_F – factor depending on the type of stress-strain state of the element. $\beta_F = 1$ for compressed elements.

k – ratio, which relates the initial modulus of elasticity of concrete E_{co} with the section modulus of concrete deformations $E_{c0} = f_{ck} / \varepsilon_{c1}$ at critical stresses $\sigma_c < f_{ck}$, $k = E_{c0} \cdot \varepsilon_{c1} / f_{ck}$.

α_s – relative value of the elasticity modulus of the reinforcement used

$$\alpha_s = E_s / 200000.$$

During plain bending at $0 < m < 1$ at $\varepsilon_s < \varepsilon_{s0,}$ dependence simplifies and reduces to the expression:

$$\eta_{\varepsilon u} = 1 + 5^3 \cdot \alpha_s \cdot \frac{k-1}{6-k} \cdot \sum_{i=1}^{n} \rho_{li} \cdot \left(\frac{x_{si}}{x}\right)^2 + 0{,}81 \cdot$$

$$\cdot \frac{\left(0{,}43 - 0{,}2(0{,}4 - m)^2\right)\left(1 - m^{3/2}\right)\left(1 - m/k\right)\sqrt{(\ln k)^{\left(1-m^{3/2}\right)}}}{1 + \left(\left(l/6 - 0{,}1m^2\right)(k-2)\ln\left(6/k - 2(0{,}1 - m)\right)\right)^2}$$

(1.18)

During the yield reinforcement in plain bending at $m = 0$, $\varepsilon_s > \varepsilon_{s0}$

$$\eta_{\varepsilon u} = 1 + 0{,}322 \cdot \sqrt{lnk} / [1 + ((k-2)\ /6 \cdot ln(6 / k - - 0{,}2))^2] \qquad (1.19)$$

9. Limit deformation of concrete in tension, in the moment of normal cracks appearance is determined by $\varepsilon_{ct0} = 0$ and $\sigma_{ct0} = 0$ according to the formula:

$$\eta_{\varepsilon_{tu}} = 1 + \frac{0{,}642 \cdot \sqrt{(lnk_t)^{1{,}4}}}{1 + \left(\frac{k_t - 2}{6} \cdot \ln\left(\frac{36}{k_t^2} - 0{,}2\right)\right)^2} \qquad (1.20)$$

10. On the limit state, the effect of concrete in tension on the bearing capacity of the reinforced concrete element, is not considered.
11. The relationship between stress and strain with physical reinforcement yield strength is taken as the Prandtl diagram or two-line diagram.

12. In the absence of physical yield strength of reinforcement, deformation diagram is described by two-line dependencies or linear-parabolic functions.

For bent concrete elements with double reinforcement (Fig. 1.4), or in the case of multi-row reinforcement equilibrium, equations in the limiting stage will change, mainly, due to the redistribution of efforts in reinforcing rods, neglecting the work of concrete in tension.

$$N_s = \frac{b_n \cdot E_s}{1/r}(\varepsilon_{s1} + \varepsilon_{cu})\sum_{i=1}^{n} \rho_{li} \cdot \varepsilon_{si} = \frac{b_n \cdot E_s}{1/\rho}(\varepsilon_{s1} + \varepsilon_{cu}) \cdot \varepsilon_{s1} \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} k_{si} \quad (1.21)$$

$$M_s = \frac{h_n \cdot E_s}{(1/r)^2}(\varepsilon_{s1} + \varepsilon_{cu})\sum_{i=1}^{n} \rho_{li} \cdot \varepsilon_{si}^2 = \frac{b_n \cdot E_s}{(1/\rho)^2}(\varepsilon_{s1} + \varepsilon_{cu}) \cdot \varepsilon_{s1}^2 \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} k_{si}^2 \quad (1.22)$$

where ρ_{li} − coefficient element section reinforcing by separate rods, $\rho_{li} = A_{si} / (b_n \cdot d)$;

ε_{si} − the relative deformation of the i-th of reinforcing bar in element section;

ε_{s1} − the current strain of the most tensed reinforcing bar;

k_{si} − coefficient characterizing the position of specific reinforcing bar in element cross-section, with respect to the most tensed, $k_{si} = x_{si} / x_{s1}$.

$k_{\rho i}$ − actuation coefficient specific of cross-sectional area of the rod to the cross-sectional area of the most tensed, $k_{\rho i} = \rho_{l1} / \rho_{II}$;

x_{s1} – the distance from the neutral axis to the most tensed reinforcing bar of concrete element.

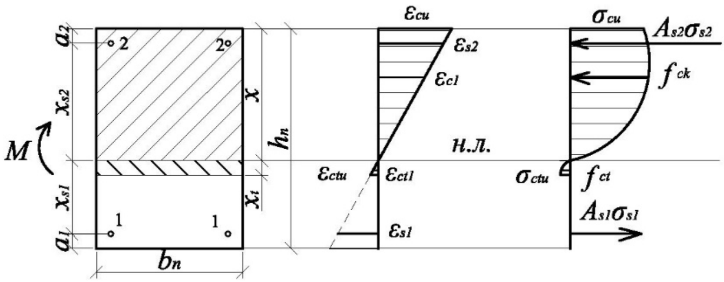

Fig 1.4. Stress-strain state of reinforced concrete bent element with double reinforcement

In the ultimate stage, deformation of the reinforcement can occur in three different schemes:

- all reinforcing bars of bent concrete element are working elastically;
- most stressed reinforcing bars are operating in the plastic stage and less tensed – are deforming elastically;
- tension in all bars reaches the yield point.

In the case of elastic deformation of reinforcing bars ($\varepsilon_{si} < \varepsilon_{s0}$), the deformation of the most tensed one, should be calculated from the expression:

$$\varepsilon_{s1} = -\varepsilon_{cu} / 2 \pm \sqrt{(\varepsilon_{cu} / 2)^2 + \delta_c \cdot \alpha_c / \left(E_s \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} \cdot k_{si} \right)} \tag{1.23}$$

In this case, bearing capacity of the element is determined by:

$$M_u = \frac{b_n}{(1/r)^2} (\delta_c \cdot \beta_c + E_s \cdot (\varepsilon_{s1} + \varepsilon_{cu}) \cdot \varepsilon_{s1}^2 \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} k_{si}^2) \tag{1.24}$$

If the flow occurs only in the part of the reinforcing bars ($\varepsilon_{s1} \dots \varepsilon_{si} \geq \varepsilon_{s0}$) and ($\varepsilon_{sm} \dots \varepsilon_{sn} < \varepsilon_{s0}$) is not present in the other, then the total force which is perceived by reinforcement of the concrete element, can be calculated by the formula:

$$N_s = \frac{b_n}{1/r} E_s (\varepsilon_{s1} + \varepsilon_{cu}) \left(\sum_{i=1}^{l} \rho_{li} \cdot \varepsilon_{s0} + \sum_{i=m}^{n} \rho_{li} \cdot \varepsilon_{s1} \right) =$$

$$= \frac{b_n}{1/r} E_s (\varepsilon_{s1} + \varepsilon_{cu}) \cdot (\varepsilon_{s0} \cdot \rho_{II} \sum_{i=1}^{l} k_{\rho i} + \varepsilon_{s1} \cdot \rho_{II} \sum_{i=m}^{n} k_{\rho i} k_{si} \tag{1.25}$$

Deformation of the most tensed reinforcing bar can be found from the equation:

$$\varepsilon_{s1}^2 + \varepsilon_{s1}(\varepsilon_{cu} + \varepsilon_{so}\varphi_{\rho 1}) + \varepsilon_{cu}\varepsilon_{so}\varphi_{\rho 1} - \delta_c \alpha_c / (E_s \varphi_{\varphi}) = 0 \tag{1.26}$$

using the expression:

$$\varepsilon_{s1} = \frac{\varepsilon_{cu} + \varepsilon_{s0}\varphi_{\rho 1}}{2} \pm \sqrt{\left(\frac{\varepsilon_{cu} + \varepsilon_{s0}\varphi_{\rho 1}}{2} \right)^2 + \delta_c \cdot \alpha_c / \left(E_s \cdot \rho_{\varphi 1} \right) - \varepsilon_{cu}\varepsilon_{s0}\varphi_{\rho 1}} \tag{1.27}$$

where the appropriate reinforcement parameters are:

$$\varphi_{\rho 1} = \rho_{\varphi} / \rho_{\varphi 1}; \ \rho_{\varphi} = \rho_{II} \sum_{i=1}^{l} k_{\rho i}; \ \rho_{\varphi 1} = \rho_{II} \sum_{i=m}^{n} k_{\rho i} k_{si} \tag{1.28}$$

$$\rho_{\varphi 0} = \rho_{II} \sum_{i=1}^{l} k_{\rho i} k_{si} ; \; \rho_{\varphi 2} = \rho_{II} \sum_{i=1}^{n} k_{\rho i} k_{si}^{2} ;$$

$$\varphi_{\rho 2} = \rho_{\varphi 0} / \rho_{\varphi 2} ; \; \varphi_{\rho 3} = \rho_{\varphi 2} / \rho_{\varphi 1} . \tag{1.29}$$

The bearing capacity of bending element takes the form:

$$M_u = \frac{b_n}{(1/r)^2} (\delta_c \cdot \beta_c + E_s \cdot (\varepsilon_{s1} + \varepsilon_{cu}) \cdot (\varepsilon_{s1} \cdot \varepsilon_{s0} \cdot \rho_{\varphi 0} + \varepsilon_{s1}^2 \rho_{\varphi 2})) \tag{1.30}$$

Upon reaching the yield stress in all reinforcing rods ($\varepsilon_{si} \geq \varepsilon_{s0}$), the deformation of the most tensed one ε_{s1} is determined by:

$$\varepsilon_{s1} = \delta_c \cdot \alpha_c / \left(E_s \cdot \varepsilon_{s0} \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} \right) - \varepsilon_{cu} \tag{1.31}$$

and load-bearing capacity of the bending element has the form:

$$M_u = \frac{b_n}{(1/r)^2} (\delta_c \cdot \beta_c + E_s \cdot \varepsilon_{s0} (\varepsilon_{s1} + \varepsilon_{cu}) \cdot \varepsilon_{s1} \cdot \rho_{II} \sum_{i=1}^{n} k_{\rho i} k_{si}) \tag{1.32}$$

1.3 Conclusions on Chapter 1:

1. Deformation model, as opposed to force, reflects the stress-strain state of reinforced concrete elements in a limit stage more accurately. However, in these models, a single general criterion of the exhaustion of the bearing capacity, has not yet been developed;

2. Generalized model of the element deformation should be able to equally reflect, both, the nature of the growth of relative deformation of materials, and a process of continuous redistribution of stresses in them. So, the real state of the reinforced concrete structure, can only be reflected when, both, stress and deformations diagrams are used in conjunction.

3. Characteristic that links the strength (*M*, *N*) and the deformational (*1/r*, *ε*) parameters can be stiffness of the element in a certain section.

CHAPTER 2 GENERAL CASE OF NONLINEAR DEFORMATION MODEL OF SPAN REINFORCED CONCRETE STRUCTURE

General case of stress state in random sections of span concrete structures involves a joint action of longitudinal and shear forces and bending moments.

Fundamentals of modern ideas about the theory of strength of concrete and reinforced concrete in the triaxial stress-deformed state were laid by M.M. Filonenko-Borodich, G.A. Heniyevym, V.M. Kyssyukom, G.A. Tyupinym, G.S. Pisarenko, A.A. Lebedev, T.A. Balan, S.F. Klovanychem, M.I. Karpenko and his students, Dei Poli, K.H. Gerstle, H.B. Kupfer and others.

The appearance of modern computers as personal computers, made it possible to solve problems with complex computational models by numerical methods.

Let us consider the reinforced concrete rectangular section rod-beam (Fig. 2.1) with constant stiffness by length, in whose computational sections, there is a general case of stress state.

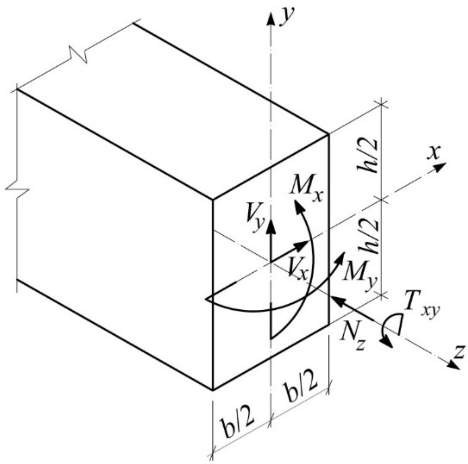

Fig. 2.1 Internal forces on rod's random normal-section in its general stress-strain state

We consider that the rod-beam is made from heavy concrete, which was hardening in normal, natural conditions. Its reinforcement is random, in the form of orthogonally directed working rods and mounting reinforcement along the axis z, transverse vertical (along the axis y) and horizontal (along the axis x) reinforcements.

Let us consider the problem of determining the bearing capacity of reinforced concrete rod considering its central compression (tension), skew bend with free or compressed torsion, the impact of structural factors and factors of external action, nonlinear properties of concrete and reinforcement, a simple proportional, small cycle constant sign and alternating loads.

The main hypotheses and conditions:

- Reinforced concrete rod element is stiff;
- Calculation sections normal to the longitudinal axis are considered;
- Relations between stresses and deformations in the concrete and reinforcement are set using full diagrams;
- the hypothesis of flat sections can be used during deformation of compressioned (tensioned) bending RC elements;
- Shear stresses in the element calculation section at its free torsion are determined in accordance with recommendations S.P. Tymoshenko [14] in the editorial of I.A. Birger and J.G. Panovko [15];
- Shear and normal stresses in rod calculation sections in its compressed torsion are determined by the decision of M.I. Bezukhov in the editorial of Y.O. Shkola [16], [17];
- Concrete and rods of longitudinal reinforcement perceive normal σ_x, σ_y, σ_z and tangent τ_{zx}, τ_{zy}, τ_{xy} stress;
- Transverse reinforcement rods perceive only tangent stresses τ_{zx} and τ_{zy}. Their distribution along the length of the rods is considered uneven;
- Phenomenological condition of strength of M.I. Karpenko and his students [18] or V.M. Kruglov [19], [20] can be accepted as a criterion of the destroying of concrete (macro cracks appearance);
- Before the appearance of macro cracks it is considered as equitable condition of strain compatibility of concrete and reinforcement. Once concrete is excluded from work and stresses in the section with crack are taken over only by reinforcement;
- Reinforcing rods are excluded from work with the appearance of yielding strains in them. As the criterion, the yielding condition of Huber-Mises-Genk [16], is taken;

- During the transition from stresses to generalized internal force factors, the procedure of numerical integration of elementary internal force factors on the entire area of calculation section, is applied. Thus, current section of rod element is conventionally divided into separate small particle elements, within which stresses are considered as equal.

According to the recommendations of G.A. Heniyev, M.I. Karpenko, S.F. Klovanych and others, the strength of concrete in the coordinate system of the main stresses σ_1, σ_2, σ_3 is described by the continuous, convex, symmetric against octahedral normal stress σ_0, and equally inclined to the said axes surface plotted by the method of M. Filonenko-Borodich using the equation

$$f(\sigma_{0c},\tau_{0c},\theta_c)=\tau_{0c}-\tau_{01c}(\sigma_{0c})\cdot\rho(\theta_c)=0 \qquad (2.1)$$

where σ_{0c},τ_{0c} – octahedral normal and tangent stresses;

ϑ – angle of stress state type;

$\rho(\theta_c)$ – interpolation function between τ_{01} $(\theta_c=60°)$ and τ_{02} $(\theta_c=0°)$, which is determined, as it is proposed by D.I.Bezushko [21], by the formula:

$$\rho(\theta_c)=\left[2a_c\cos\theta_c+b_c\sqrt{a_c\left(4\cos^2\theta_c-1\right)+b_c^2}\right]\Big/\left(4a_c\cos^2\theta_c+b_c^2\right) \qquad (2.2)$$

where $a_c=1-c_c^2,\ \ b_c=2c_c-1,\ \ c_c=\tau_{o2c}/\tau_{o1c}$.

Relation between octahedral stresses at angles of stress state type $\theta_c=60°$ та $\theta_c=0°$ can be shown as:

$$\sigma_{oc}=A_1\tau_{o1c}^2+B_1\tau_{o1c}+C_1,\quad \sigma_{oc}=A_2\tau_{o2c}^2+B_2\tau_{o2c}+C_1 \qquad (2.3)$$

Factors A_1, A_2, B_1, B_2, C_1 are achieved by the way of “binding” of characteristic points in the surface of concrete strength. Using the experimental relations of M.M. Bondarenko and V.I. Kolchunov, it is proposed to determine them by these simplified formulas in author’s interpretation:

$$\begin{aligned}
&A_1=4{,}14/\left(f_{ck}-f_{ctk}\right);\\
&B_1=\left(5{,}38f_{ck}^2+f_{ck}f_{ctk}-6{,}38f_{ck}^2\right)\Big/\left[4{,}24\left(f_{ck}-f_{ctk}\right)^2\right];\\
&A_2=\left(4{,}09f_{ck}-4{,}16f_{ctk}\right)\Big/\left(1{,}20f_{ck}^2-2{,}20f_{ck}f_{ctk}+f_{ctk}^2\right);\\
&B_2=\left(4{,}46f_{ck}^2-2{,}04f_{ck}f_{ctk}-0{,}73f_{ctk}^2\right)\Big/\left(4{,}32f_{ck}^2-7{,}92f_{ck}f_{ctk}+3{,}60f_{ctk}^2\right);\\
&C_1=-H=-\left(0{,}82f_{ck}f_{ctk}\right)\Big/\left(f_{ck}-f_{ctk}\right),
\end{aligned} \qquad (2.4)$$

where f_{ck}, f_{ctk} – characteristic (design value f_{cd}, f_{ctd}) concrete strength, according, for compression and tension. With the formula (2.1), the surface of concrete strength can, uniquely, be described, because it consists of coefficients (2.4) of five independent parameters of (its) strength, which correspond to individual cases of stress state:

- uniaxial compression $R_c = f_{ck}$ and tension $R_p = f_{ctk}$,
- biaxial compression $R_{2c} \cong 1,2R_c = 1,2 f_{ck}$ and tension $R_{2p} \cong R_p = f_{ctk}$,
- triaxial uniform tension $R_{3p} = H \cong \left(0,82R_c \cdot R_p\right) / \left(R_c - R_p\right)$.

Angle of stress state type in rod concrete which is considered, can be determined by M.I. Bezukhov [16] considering $\sigma_x = \sigma_y = 0$

$$\theta_c = \frac{1}{3}\arccos\left(\frac{3\sqrt{3D_3}}{2\sqrt{D_2^3}}\right) = \frac{1}{3}\arccos\left[\frac{\sqrt{\sigma_{zc}\left[2\sigma_{zc}^2 + 9\left(\tau_{xyc}^2 + \tau_{zyc}^2 + \tau_{zxc}^2\right)\right]}}{2\sqrt{\left(\sigma_{zc}^2/3 + \tau_{xyc}^2 + \tau_{zyc}^2 + \tau_{zxc}^2\right)^3}}\right] \quad (2.5)$$

where D_2, D_3 - second and third invariants of stress deviator.

Considering (2.1), (2.3) we will get:

$$\sigma_{oc} = \frac{A_1}{\rho^2(\theta_c)}\tau_{oc}^2 + \frac{B_1}{\rho(\theta_c)}\tau_{oc} + C_1 \quad (2.6)$$

Limit values of concrete strength (on the "surface" of strength) in the form of $\widehat{\sigma}_{oc}$ and $\widehat{\tau}_{oc}$ are determined by solution of equation system:

$$\begin{cases} \widehat{\tau}_{oc} - \tau_m = m_\sigma\left(\widehat{\sigma}_{oc} - \sigma_m\right); \\ \widehat{\sigma}_{oc} = \dfrac{A_1}{\rho^2(\theta_c)}\widehat{\tau}_{oc}^2 + \dfrac{B_1}{\rho(\theta_c)}\widehat{\tau}_{oc} + C_1, \end{cases} \quad (2.7)$$

where σ_m and τ_m – stresses on previous load level (at simple proportional loading $\sigma_m = \tau_m = 0$);

m_σ – factor that characterized stress-strain state of concrete

Huber-Mises-Hencky condition of reinforcing steel yield [16] at $\sigma_x = \sigma_y = 0$ looks like:

$$\sigma_{zs}^2 + 3\tau_{xys}^2 + 3\tau_{zxs}^2 + 3\tau_{zys}^2 = \tilde{f}_{yd}^2 \quad (2.8)$$

where $\tilde{f}_{yd}^2$ – calculation strength of reinforcement on the yield limit considering its decrease because of complex stress state compared with central tension-compression.

According to recommendations of M.I. Karpenko [18] concrete deformation diagram at compression (tension) can be shown as:

$$\varepsilon_b = \frac{\sigma_b}{E_b^0 \nu_b} = \frac{\sigma_c}{E_{c0} \varsigma_c} = \varepsilon_c \tag{2.9}$$

where $\varepsilon_b = \varepsilon_c$ – relative linear concrete deformations;

$\sigma_b = \sigma_c$ – normal stresses in concrete;

$E_b^0 = E_{c0}$ – initial concrete modulus of elasticity;

$\nu_b = \zeta_c$ – factor of secant concrete modulus of elasticity change.

Deformation relations for concrete, which is, in complex stress-strain state, advisable to formulate as a link between octahedral stresses and deformations. With next hypotheses considered as equitable:

- link between octahedral stresses τ_{oc} and shears on octahedral planes γ_{oc} is nonlinear: $\tau_{oc} = G_c(\gamma_{oc}) \cdot \gamma_{oc}$, де $G_c(\gamma_{oc})$ - secant (octahedral) concrete shear modulus;

- link between octahedral normal stresses σ_{oc} and average deformations ε_{oc} is also nonlinear and can be shown as: $\sigma_{oc} = K(\gamma_{oc}) \cdot (\varepsilon_{oc} - \rho_c \gamma_{oc}^2)$, where ρ_c – modulus of dilatation (by G.O. Heniyev [22] – g_{oc}); $K(\gamma_{oc})$ – modulus of volume deformations.

For determination of secant modules analogically to the hypothesis of "single curvature of deformation", it is advisable to use the hypothesis of S.F. Klovanych and D.I. Bezushko, according to which the form of link between stresses and deformations is not depending on the stress state type, i.e. link between τ_{oc} and γ_{oc} can be accepted similar to uniaxial compression and for determination of secant shear modulus the relation EKB (Fig. 2.2) $G_c(\gamma_{oc}) = G_{oc} \cdot f(\gamma_o)$ proposed by Siense can be accepted. Here:

$$f\left(\gamma_{oc}\right)=\frac{1}{1+A\eta+B\eta^{2}+C\eta^{3}} \tag{2.10}$$

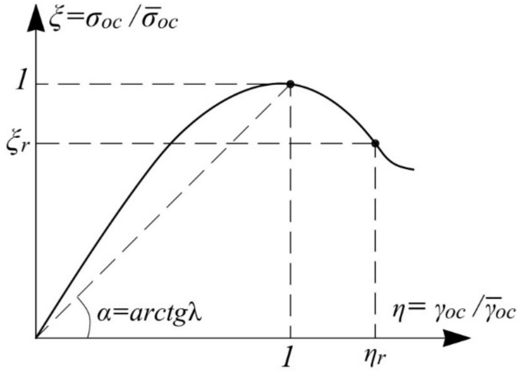

Fig. 2.2 Concrete deforming diagram at triaxle stress state

where:

$C=\lambda(1-\xi_r)/\left[\xi_r(\eta_r-1)^2-1/\eta_r\right]$; $B=1-2C$; $A=C+\lambda-2$; $\xi_r=\bar{\sigma}_r/f_{ck}\approx 0{,}85$ i

$\eta_r=\gamma/\bar{\gamma}_r\approx 1{,}41$; $\xi=\sigma_{oc}/f_{ck}$; $\eta=\gamma_{oc}/\bar{\gamma}_{oc}$; $\lambda=\xi/\eta$;

initial displacement module

$$G_{oc}=G_{c0}=E_{c0}/\left[2\left(1+v_c\right)\right];\ \sigma_{oc}=\left(\sigma_{xc}+\sigma_{yc}+\sigma_{zc}\right)/3;\ \varepsilon_{oc}=\left(\varepsilon_{xc}+\varepsilon_{yc}+\varepsilon_{zc}\right)/3;$$

$$\tau_{oc}=1/3\sqrt{\left(\sigma_{xc}-\sigma_{yc}\right)^2+\left(\sigma_{zc}-\sigma_{yc}\right)^2+\left(\sigma_{zc}-\sigma_{xc}\right)^2+6\left(\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)};$$

$$\gamma_{oc}=2/3\sqrt{\left(\varepsilon_{xc}-\varepsilon_{yc}\right)^2+\left(\varepsilon_{zc}-\varepsilon_{yc}\right)^2+\left(\varepsilon_{zc}-\varepsilon_{xc}\right)^2+3/2\left(\gamma_{xyc}^2+\gamma_{zyc}^2+\gamma_{zxc}^2\right)}.$$

Considering $\sigma_{xc}=\sigma_{yc}=0$ for considered rod: $\sigma_{oc}=\sigma_{zc}/3$; $\varepsilon_{oc}=\varepsilon_{zc}/3$; $\tau_{oc}=1/3\sqrt{2\sigma_{zc}^2+6\left(\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)}$; $\gamma_{oc}=2/3\sqrt{2\varepsilon_{zc}^2+3/2\left(\gamma_{xyc}^2+\gamma_{zyc}^2+\gamma_{zxc}^2\right)}$. It is recommended to determine limit (maximum possible) shears $\bar{\gamma}_r$ on octahedral planes by regression equation of D.I. Bezushko [21], received as a result of processing of known A.V. Yashin and M.D. Kotsovos experimental data of triaxle compression:

$$\bar{\gamma}_r=7{,}97\left(\tau_{oc}/f_{ck}\right)^2+15{,}22\left(\tau_{oc}/f_{ck}\right)-3{,}713 \tag{2.11}$$

Dilatation modulus of concrete by Heniyev H.O. [22]:

$$\rho_c = g_{oc} = -\theta_c / \tilde{A}_c^2 = -\left(\varepsilon_{xc} + \varepsilon_{yc} + \varepsilon_{z\tilde{n}}\right) G_{oc} / 4 f_{bk} \qquad (2.12)$$

where $\theta_c, \tilde{A}_c$ – respectively limiting volume deformation and intensity of concrete shear deformation at free shear;

f_{bk} – characteristic (design f_{bd}) value of limit stresses of coupling which approximately is: $f_{bd} = R_{b,sh} = 0,7\sqrt{R_b R_{bt}}$ by V.M. Baykov.

Modulus of volume deformations is determined analogically: $K_c(\gamma_{oc}) = K_{oc} \cdot f(\gamma_{oc})$, where $K_{oc} = \dfrac{E_{c0}}{1-2v_c}$ – initial modulus of volume deformations.

Hence the secant modulus of elasticity E_c and coefficient of transverse deformations v_c of complex stressed concrete by M.I Karpenko [18], are determined by:

$$E_c = 3K_c(\gamma_{oc})G_c(\gamma_{oc}) / \left[G_c(\gamma_{oc}) + K_c(\gamma_{oc})\right],$$
$$v_c = \left[K_c(\gamma_{oc}) - 2G_c(\gamma_{oc})\right] / \left\{2\left[G_c(\gamma_{oc}) + K_c(\gamma_{oc})\right]\right\}. \qquad (2.13)$$

Similarly, we can get the formula for secant modulus of elasticity at shear for reinforcement steel and relations for diagram of its shear:

$$G_s = \frac{E_{sk}\vartheta_s}{[2(1+v_s)]}; \; \tau_s = \frac{E_{sk}\vartheta_s}{[2(1+v_s)]}\gamma_s \qquad (2.14)$$

where v_s – factor of secant modulus of elasticity change.

Axial deformation in rods of transverse reinforcement and relative angle deformation in adjacent concrete can be calculated by O.F. Yaremenko and Yu.O. Shkola [23]:

$$\varepsilon_{sw}^* = \gamma_c^* = \gamma_c\left[1 + d_{sw}E_{sw}\nu_{sw}(1+\nu_c)/(2l_{sw}E_{c0}\upsilon_c)\right]^{-1} \qquad (2.15)$$

Joint work of longitudinal and transverse reinforcement at reinforced concrete calculations is considered by reducing the design value of yield limit of longitudinal reinforcement by V.M. Baykov and Yu.O. Shkola [31] with reducing factor k_1:

$$\tilde{f}_{yd} = f_{yd}\sqrt{1-3s^2k_1\left(ctg^2\alpha/l^2_{sw,x} + ctg^2\beta/l^2_{sw,y}\right)\Big/\left[4\left(1+v_s\right)^2\right]}, \qquad (2.16)$$

where the value of reducing factor k_1 = 0,08…0,10.

2.1 Calculation element cross section

Concrete part of rod cross section is conditionally divided on small parts of rectangle form (Fig. 2.3), dimensions of which conform to the fineness of largest concrete fraction.

Each of these particles is assigned a serial number. For each *n* particle of concrete in calculation cross section, there are fixed coordinates of its centroid relative to the centre of symmetry axes of section x_{cn}, y_{cn}, area A_{cn}, characteristic concrete compression strength f_{ck}, tension strength f_{ctk}, initial modulus of elasticity E_{c0}. Poisson ratio v_c=0,2.

Location of rods of longitudinal reinforcement is taken discrete. Each longitudinal reinforcement rod is assigned own number *j*, indicating its diameter d_{sj}, location of centroid relative to the centre of symmetry axes of elements section x_{sj}, y_{sj}, characteristic values of strength on yield limit f_{ykj} (or $f_{0,2kj}$), relative reinforcement steel deformations ε_{uk}, initial modulus of elasticity E_{skj} and reinforcement grade. The Poisson ratio v_s = 0,25.

Location of transverse reinforcement in the plane of calculation section is taken as discrete too.

Horizontal and vertical rods of transverse reinforcement (stirrups) are conditionally divided on separate areas, each of them assigned number *i*, are arranged with concrete surface area A_{cswi} and coordinates of its centroid in the plane of calculation cross section x_{swi}, y_{swi} relative to symmetry axes.

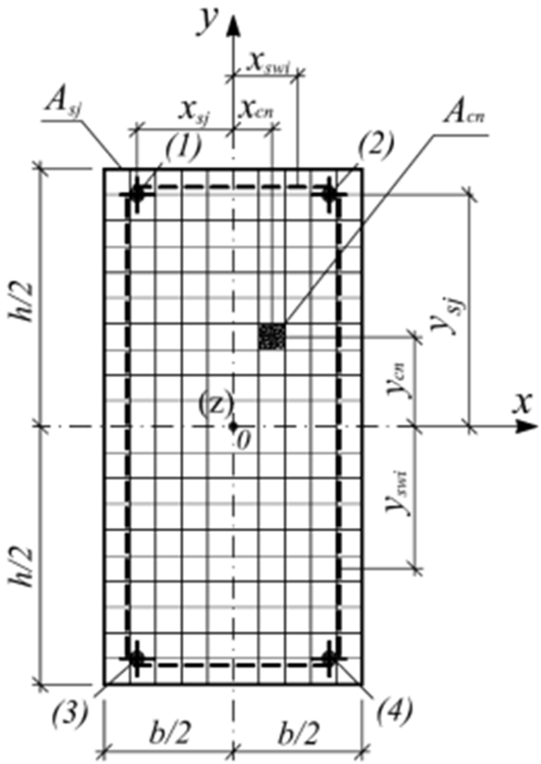

Fig. 2.3 Particle components of the calculation cross section of the rod

Following vakues are set: characteristic value of strength on the limit of yield f_{ywk}, characteristic value of tension strength f_{twk}, modulus of elasticity E_{sw}, the Poisson ratio $v_{sw}=0,25$, characteristic value of relative deformations ε_{uwk}, limit or level of elasticity and transverse reinforcement grade.

On the element length the transverse reinforcement is considered as distributed layer of area per unit length on its edge:

$$A_{zswi} = \pi d_{swi}^{2} / (4 s_i) \tag{2.17}$$

where s_i – step of transverse rods on longitudinal direction.

Equilibrium equations.

$$N_z = \sum_{n=1}^{k} A_{cn}\sigma_{zcn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj}$$

$$M_x = \sum_{n=1}^{k} A_{cn}\sigma_{zcn}Y_{cn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj}Y_{sj}$$

$$M_y = \sum_{n=1}^{k} A_{cn}\sigma_{zcn}X_{cn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj}X_{sj}$$

$$V_x = \sum_{n=1}^{k} A_{cn}\tau_{zxcn} + \sum_{j=1}^{m} A_{sj}\tau_{zxsj} + \sum_{i=1}^{l_{xsw,1-2,3-4}} A_{xswi}\sigma_{xswi} \tag{2.18}$$

$$V_y = \sum_{n=1}^{k} A_{cn}\tau_{zycn} + \sum_{j=1}^{m} A_{sj}\tau_{zysj} + \sum_{i=1}^{l_{ysw,1-4,2-3}} A_{yswi}\sigma_{yswi}$$

$$T_{xy} = \sum_{n=1}^{k} A_{cn}\left(\tau_{zycn}X_{cn}^{tor} - \tau_{zxcn}Y_{cn}^{tor}\right) + \sum_{j=1}^{m} A_{sj}\left(\tau_{zysj}X_{sj}^{tor} - \tau_{zxsj}Y_{sj}^{tor}\right) +$$

$$+ \sum_{i=1}^{l_{xysw,1...4}} A_{swi}(\sigma_{yswi}X_{swi}^{tor} - \sigma_{xswi}X_{swi}^{tor})$$

where σ_{zcn} – normal stresses in *n* particle of concrete section;
σ_{zsj} – normal stresses in *j* longitudinal rod;
τ_{zxcn}, τ_{zycn} – tangent stresses in *n* particle of concrete section;
τ_{zxsj}, τ_{zysj} – tangent stresses in *j* longitudinal rod;
σ_{xswi}, σ_{yswi} – normal stresses in on *i* piece of, accordingly, horizontal and vertical transverse reinforcement.

Normal and tangent stresses:

$$\sigma_{zml} = E_{ml}\zeta_{zml}\varepsilon_{zml}\,;\ \tau_{zxml} = G_{ml}\vartheta_{zxml}\gamma_{zxml}\,;$$
$$\tau_{zyml} = G_{ml}\vartheta_{zyml}\gamma_{zyml}\,;\ \tau_{xyml} = G_{ml}\vartheta_{xyml}\gamma_{xyml} \tag{2.19}$$

where ζ– factor of secant modulus of elasticity of concrete E_{ml} change;

ϑ – factor of secant shear modulus G_{ml} change;

m = c for particles of concrete section; *m = s* for rods of longitudinal reinforcement; *m = sw* for rods of transverse reinforcement;

l – number of concrete or longitudinal reinforcement rod particle;

i – number of transverse reinforcement rod particle.

Generalized linear and angular deformations determined by the hypothesis of flat sections, H. Han solutions of theory of elasticity [24] at bending, Y.O. Shkola functions of stresses distribution at constrained torsion and Saint-Venant functions at free torsion:

$$\begin{aligned} \varepsilon_{zml} &= \varepsilon_0 + \chi_x X_{ml} + \chi_y Y_{ml} + \beta_z \theta_z \varphi(X_{ml}^{tor}, Y_{ml}^{tor}) \\ \gamma_{zxml} &= K_x g_{xml} + K_y h_{yml} + \theta_z f_{zxml} \\ \gamma_{zyml} &= K_y g_{yml} + K_x h_{xml} - \theta_z f_{zyml} \\ \gamma_{xyml} &= -\theta_z f_{xyml} \end{aligned} \tag{2.20}$$

where ε_0 – axial relative deformation of element along axis *z*;

χ_x, χ_y – bending curvatures in planes of moments M_x, M_y accordingly;

K_x, K_y, – shear curvatures in planes of shear forces V_x, V_y, accordingly;

θ_z – relative (per unit length) angle of twist if rod unit length (rad/m);

$\varphi\left(X_{ml}^{tor}, Y_{ml}^{tor}\right)$ - Saint-Venant center of twist related torsion function;

β_z – factor of section deplanation, which, for constrained torsion, is determined by the formula: $\beta_z = \eta e^{-\eta z}$,

where *η* – Yu.O. Shkola compression factor [17];

z – distance along element axle to the closest rigid fixing. At free torsion $\beta_z = 1$

g_{xml}, g_{yml}, h_{xml}, h_{yml} – H. Han functions of angle deformation at transverse bending distribution;

$f_{zxml} = \dfrac{\tau_{zxml}}{(\theta_z \cdot G_{ml})}$, $f_{zyml} = \dfrac{\tau_{zyml}}{(\theta_z \cdot G_{ml})}$, $f_{xyml} = \dfrac{\tau_{xyml}}{(\theta_z \cdot G_{ml})}$ – function of tangent stresses at free and constrained torsion distribution.

General physical correlations:

Developing ideas [25] with considering the shear force action, general physical correlations for calculation cross section of reinforced concrete rod took the form:

$$\begin{Bmatrix} N_z \\ M_y \\ M_x \\ V_x \\ V_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & D_{16} \\ D_{21} & D_{22} & D_{23} & 0 & 0 & D_{26} \\ D_{31} & D_{32} & D_{33} & 0 & 0 & D_{36} \\ 0 & 0 & 0 & D_{44} & D_{45} & D_{46} \\ 0 & 0 & 0 & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \chi_y \\ \chi_x \\ K_x \\ K_y \\ \theta \end{Bmatrix} \tag{2.21}$$

or $\{N\}=[D]\{\varepsilon\}$,

where D_{11} – axial stiffness of element:

$$D_{11} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} / \psi_{sj} \tag{2.22}$$

D_{22}, D_{33} – bending stiffnesses in planes *zox*, *zoy*:

$$D_{22} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} X_{cn}^2 + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj}^2 / \psi_{sj} \tag{2.23}$$

$$D_{33} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} Y_{cn}^2 + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} Y_{sj}^2 / \psi_{sj} \tag{2.24}$$

D_{23} – stiffness of mutual influence of bending in two planes:

$$D_{23} = D_{32} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} X_{cn}^2 Y_{cn}^2 + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj}^2 Y_{sj}^2 / \psi_{sj} \tag{2.25}$$

D_{12}, D_{13} – stiffnesses of influence of longitudinal force on bending and bending moments on element elongation:

$$D_{12} = D_{21} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} X_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj} / \psi_{sj} \tag{2.26}$$

$$D_{13} = D_{31} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} Y_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} Y_{sj} / \psi_{sj} \tag{2.27}$$

D_{44}, D_{55} – stiffnesses of shear in planes *zox*, *zoy* from shear forces action:

$$D_{44} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zxcn} g_{xcn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zxsj} g_{xsj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,x}} A_{swi} E_{sw} \zeta_{xswi} g_{xswi,c} \quad (2.28)$$

$$D_{55} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zycn} g_{ycn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zysj} g_{ysj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,y}} A_{swi} E_{sw} \zeta_{yswi} g_{yswi,c} \quad (2.29)$$

D_{45}, D_{54} – stiffness of mutual influence of bending in planes *zox*, *zoy*:

$$D_{45} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zxcn} h_{xcn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zxsj} h_{xsj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,x}} A_{swi} E_{sw} \zeta_{xswi} h_{xswi,c} \quad (2.30)$$

$$D_{54} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zycn} h_{ycn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zysj} h_{ysj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,y}} A_{swi} E_{sw} \zeta_{yswi} h_{yswi,c} \quad (2.31)$$

D_{16}, D_{26}, D_{36} – stiffnesses of influence of torsion moment T_{xy} on elongation and bending curvatures in planes *zox*, *zoy*, longitudinal force *N* and bending moments M_x, M_y on shear in plane *xoy*:

$$D_{16} = D_{61} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} \beta_z \varphi\left(X_{cn}^{tor}, Y_{cn}^{tor}\right) + \sum_{j=1}^{m} A_{sj} E_s \zeta_{zsj} \beta_z \varphi\left(X_{sj}^{tor}, Y_{sj}^{tor}\right) / \psi_{sj} \quad (2.32)$$

$$D_{26} = D_{62} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} X_{cn}^{tor} \beta_z \varphi\left(X_{cn}^{tor}, Y_{cn}^{tor}\right) + \sum_{j=1}^{m} A_{sj} E_s \zeta_{zsj} X_{sj}^{tor} \beta_z \varphi\left(X_{sj}^{tor}, Y_{sj}^{tor}\right) / \psi_{sj} \quad (2.33)$$

$$D_{36} = D_{63} = \sum_{n=1}^{k} A_{cn} E_{c0n} \zeta_{cn} Y_{cn}^{tor} \beta_z \varphi\left(X_{cn}^{tor}, Y_{cn}^{tor}\right) + \sum_{j=1}^{m} A_{sj} E_s \zeta_{zsj} Y_{sj}^{tor} \beta_z \varphi\left(X_{sj}^{tor}, Y_{sj}^{tor}\right) / \psi_{sj} \quad (2.34)$$

D_{46}, D_{56}, – stiffnesses of influence of torsion moment T_{xy} on shear in planes *zox*, *zoy* and shear forces V_x, V_y on shear in plane *xoy*:

$$D_{46} = D_{64} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zxcn} f_{zxcn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zxsj} f_{zxsj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,x}} A_{swi} E_{sw} \zeta_{xswi} h_{zxswi,c} \quad (2.35)$$

$$D_{56} = D_{65} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{zycn} f_{zycn}}{[2(1+v_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zysj} f_{zysj}}{[2(1+v_s)]} + \sum_{i=1}^{l_{sw,y}} A_{swi} E_{sw} \zeta_{yswi} h_{zyswi,c} \quad (2.36)$$

D_{66}, – stiffness at rod torsion in plane *xoy*:

$$D_{66} = \sum_{n=1}^{k} \frac{A_{cn} E_{c0n} \vartheta_{xycn}}{[2(1+v_c)]} \left(f_{zycn} X_{cn}^{tor} - f_{zxcn} X_{cn}^{tor} \right) + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{xysj}}{[2(1+v_s)]} \left(f_{zysj} X_{sj}^{tor} - f_{zxsj} X_{sj}^{tor} \right) +$$
$$+ \sum_{i=1}^{l_{sw,y}+l_{sw,x}} A_{swi} E_{sw} \zeta_{xyswi} \left(f_{zyswi} X_{swi}^{tor} - f_{zxswi} X_{swi}^{tor} \right)$$

(2.37)

where ψ_{sj} – V.I. Murashev factor:

$$\psi_{sj} = 1 - \omega \sigma_{sj,cr} / \sigma_{sj} \qquad (2.38)$$

where $\sigma_{sj,cr}$ – stresses in в j rod at the moment of crack appearance;
σ_{sj} – current stress in j rod of longitudinal reinforcement;
ω – factor of completeness of diagrams of tensioned concrete,
$\omega = 0,7\cdot$

It is believed that physical correlations (2.22)…(2.38) are valid on all stages of stress-strain state of rod reinforced concrete elements of rectangle section at their simple proportional loading.

On any loading stage the deformation vector

$$\{\varepsilon\} = [D]^{-1}\{N\} \qquad (2.39)$$

The flowchart of algorithm of bearing capacity of reinforced concrete rods of rectangle section with arbitrary orthogonal reinforcement determining [26] is shown in Fig. 2.4.

Algorithm consists of block of source data input, main part, auxiliary routines of conditions of load vector increasing check, bearing capacity exhaustion and calculations printing block.

On every loading stage the calculation is performed by performance of some quantity of iterations to the time, while the accuracy of all components of the deformation vector would not meet some predetermined value, i.e. until execution of condition:

$$(\{\varepsilon\}_n - \{\varepsilon\}_{n-1}) / \{\varepsilon\}_n < \{\eta\} \qquad (2.40)$$

where $\{\varepsilon\}_n$ – deformation vector, calculated on n iteration;

$\varepsilon\}_{n-1}$ – the same on previous *n-1* iteration;

$\{\eta\}$ – accuracy vector, combined of pre-set accuracies for each component of deformation vector.

Consistently increasing the vector of given ratio forces acting in rod, the bearing capacity of reinforced concrete rod can be determined. As a limit load maximum force vector $\{N\}$ is applied, at which the system of equations (2.21) has a solution, i.e. matrix determinant $[D]$ is not zero (with some pre-set tolerance η_{det}) or reinforced concrete road bearing capacity is considered exhausted if

$$\det[D] < \eta_{\det} \qquad (2.41)$$

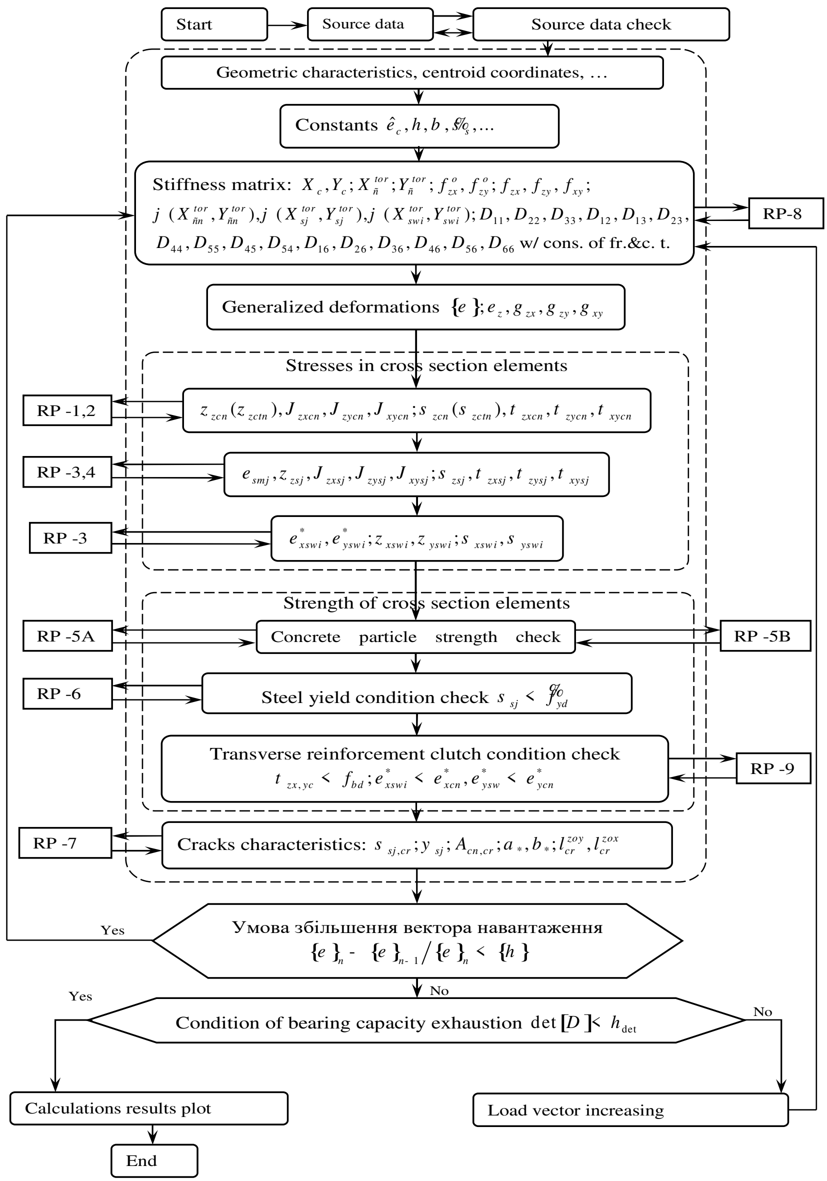

Fig. 2.4 The flowchart of algorithm of bearing capacity of reinforced concrete rod at complex stress determining

2.2 Conclusions on chapter 2:

1. Accepted in general case, nonlinear deformational model of rod construction allows to form the work union of reinforced concrete mechanics, that considers features of mutual work of concrete and reinforcement on all stages, including destruction. It can be used at design or strengthening of beams, cross-bars, columns and elements of girder frames of rectangle cross section and at check of bearing capacity of existing rod reinforced concrete structures, which are working in conditions of complex stress-strain state, including small cycle permanent sign load;

2. Prediction of cracking on the faces of reinforced concrete rod and checking concrete strength advisable to perform by three-level criterion of V.M. Kruglov or five parametric criterion of M.I. Karpenko, his students and followers;

3. Crack appearance in concrete is considered by excluding from calculation that particles of concrete, which spatial stress-strain state does not satisfy criterions of strength.

CHAPTER 3 RESISTANCE MODEL OF PREFABRICATED CONCRETE ELEMENTS FOR THE PERMANENT CYCLIC ACTION OF THE TRANSITIONAL FORCES OF HIGH LEVELS

In the current design standards, the calculation of durability in bay reinforcement concrete structures is performing under the assumption of concrete elastic work. Calculation of sloping sections is performed under the assumption that main tension stresses, which appear on the level of centroid of transformed section, should be fully carried by transverse reinforcement at stresses in them, which are equal to calculation resistance of transverse reinforcement f_{sw}, multiplied by condition load effect factor γ_{sw}, in elements without transverse reinforcement – by concrete at stresses in it, which are equal to concrete calculation tension resistance f_{ctd}, multiplied by an appropriate condition load effect factor γ_c.

Such calculation approach contradicts the real work character of inelastic work of reinforced concrete elements, and does not display reinforced concrete behaviour features in the zone of transverse forces actions at cycling loads, does not display real stress-strain state, does not consider the ambiguity of transverse forces perception by different elements at different shear bays and character of fatigue destruction crack appearance and propagation, does not consider or take into account indirectly the influence of a number of constructive factors and factors of external action, which ultimately leads to a significant difference in the calculation and experimental data.

Researchers' main attention is paid to the study of durability and stress-strain state of normal sections of elements which are bent, to the durability of concrete and reinforcement and their deformability at second loads. During these researches a lot of experimental data has been accumulated, and a number of practical methods of normal section calculation in the zone of structures pure bending have been proposed.

Despite the large number of experimental and theoretic researches of reinforcement concrete elements resistance to the transverse forces action at static loads, the problem of reinforced concrete resistance to the second loads action remain mainly unexplored.

Theoretical researches of development of physical models of bending reinforced concrete elements fatigue resistance to the series action of transverse forces and calculation methods on its base, are almost missing. Therefore, the development of physical models of fatigue resistance and destruction of near support parts of beams, which correctly shows their real work considering real element concrete and reinforcement deforming at different shear spans and corresponding methods of their calculation, has just started.

Complexity of results of previous researches in the field of reinforced concrete elements resistance to the action of transverse forces at static loads, and durability of concrete and reinforcement in normal sections of reinforced concrete structures considering their real deforming, had created objective conditions to design the science basics of the theory of fatigue resistance of their near support places to secondary action of transverse load.

3.1 Model of reinforced concrete elements resistance with no shear span reinforcement

O.S. Zalesov, Yu.A. Klimov, I.T. Mirsayapov [27] and others, identify main forms of fatigue destruction of reinforced concrete elements depending on the relative shear span c_0 / h_0: elements with zero shear span ($c_0 / h_0 = 0$), with small shear span ($c_0 / h_0 \leq 1{,}2$), with middle shear span ($1{,}2 < c_0 / h_0 \leq 2$) and with large shear span ($c_0 / h_0 > 2$). Their researches, using thermal imager, have shown that in elements with $c_0 / h_0 \leq 1{,}2$ occur local stress strips between points of applying of concentrated force and beam support, within which fatigue destruction occurs. With the subsequent decrease of relative shear span, limit state occurs, when $c_0 / h_0 = 0$ i $M_{max} = Q_{max} = 0$, i.e. force action lines and reactions converge and local compression occurs, which can, also, be included into general system of reinforced concrete elements resistance to the transverse forces action.

It is obvious that for each of indicated groups, a method of strengths and durability calculation of near support places of reinforced concrete elements considering the value of shear span, should be developed. Contrary to the normative method, it should be based on physical model of their fatigue resistance to mutual action of transverse forces and bending moments. Construction of the settlement machine is carried out considering real stress state of the elements, all internal forces and influence of the most significant factors on their fatigue resistance. Durability calculation is carried out by classic scheme: on the base of physical models, internal forces and stresses in specific sections are calculated; determining the limits of concrete durability and reinforcement durability, its anchoring; conditions of endurance are checked.

In contrast to technique standards, stresses and factors of stresses asymmetry in concrete and reinforcement, are determined considering their change during cycle loading as a result of extension of deformations of vibro-creep of compressed concrete in conditions of limited deformation capability. Limits of durability (objective fatigue strength at cycle and small cycle load) of concrete and reinforcement should be determined by appropriate criterions of fatigue strength considering appearance and extension of fatigue micro- and macro-cracks, nonelastic properties of concrete, real deformation modes of concrete and reinforcement as a part of a structure.

Despite the number of theoretic and experimental researches of concrete and reinforced concrete resistance at local static load action, in literature, available to author, there was no information about concrete and reinforced concrete behaviour at local action of repeated load. Thus, in domestic design standards there are no clear recommendations about the calculation of reinforced concrete elements at local cycle compression load, particularly with zero shear span.

Relying on researches, carried out by V.G. Donchenko, O.S. Zalesov, V.G. Kvasha, M.M. Kholmyanskiy, I.T. Mirsayapov [27] and other, physic model of fatigue resistance of concrete at such load can be represented as following: at local compression of concrete element, cross-directed force flow occurs between load areas, limited by dimensions of load areas. Stress state inside this flow is heterogeneous, since after applying the repeated load on flat element under load areas of limited width, friction forces between these areas and concrete surface appear, through which, impacted volumes in the form of wedge (Fig. 3.1, a) with edges appear in concrete, inclined to the area of load transfer with an angle that is equal to the angle of inner shear of concrete φ (Fig. 3.1, b), and inside the wedge stress state "compression-compression" ($\sigma_{1c}^{max}(t),\ \sigma_{2c}^{max}(t)$) is forming. Wedge displacements as a solid and its "wedging" in environing concrete cause appearance of thrust and, as a result, splitting (tensile) stresses $\sigma_{2ct}^{max}(t)$ between vertexes of impacted wedges. Along the edges of wedge the condition of pure shear is realized and tangential stresses appear $\tau_{12}^{max}(t)$. As a result of pressure of these wedges of impact as a solid on environing concrete, compression stresses appeared in it too $\sigma_{1c}^{max}(t)$ (Fig. 3.1, c). The compression core with width of l_{ef}, less than width of load area l_{loc}, appears in the middle zone, between vertexes of impacted wedges in elements with dimensions $H \leq 1{,}5L$ and $l_{loc} / H > 0{,}2$ by results of B.S. Sokolov researches [28].

At the cycle load of concrete, fatigue destruction and nonlinear deformations of vibro-creep, are characterized by the appearance and extension of cracks of normal rupture. At the initial stage of loading, after the excess of the average compressive stresses $\sigma_{1c}^{max}(t_0)$ of the initial level of formation of microcracks in concrete elements, there are microscopic fractures of the concrete separation. With the increase of the load level or the number of its cycles, the microcracks of the separation in the middle of the concrete element between the tops of the sealing wedges are first developed, which are then connected, forming a fatigue macro crunch, parallel or with a slight inclination to the line of external compressive forces (the main compressive stresses). When the total length $l(t)$ of fatigue macro crack of detachment reaches critical length l_{cr}, and the dynamic development of main detachment macro crack starts, that, in some circumstances, leads to ultimate destruction of compressed concrete element. Research of the destruction surface of concrete elements shows, that the ultimate destruction occurs due to shear by inclined areas under load areas, i.e. by maximum tangential stresses. Wherein, the volume is separated from the main mass wedge. It is “impact wedge”. The surface of wedge is uneven. Concrete is not destroyed inside the wedge and beyond its borders.

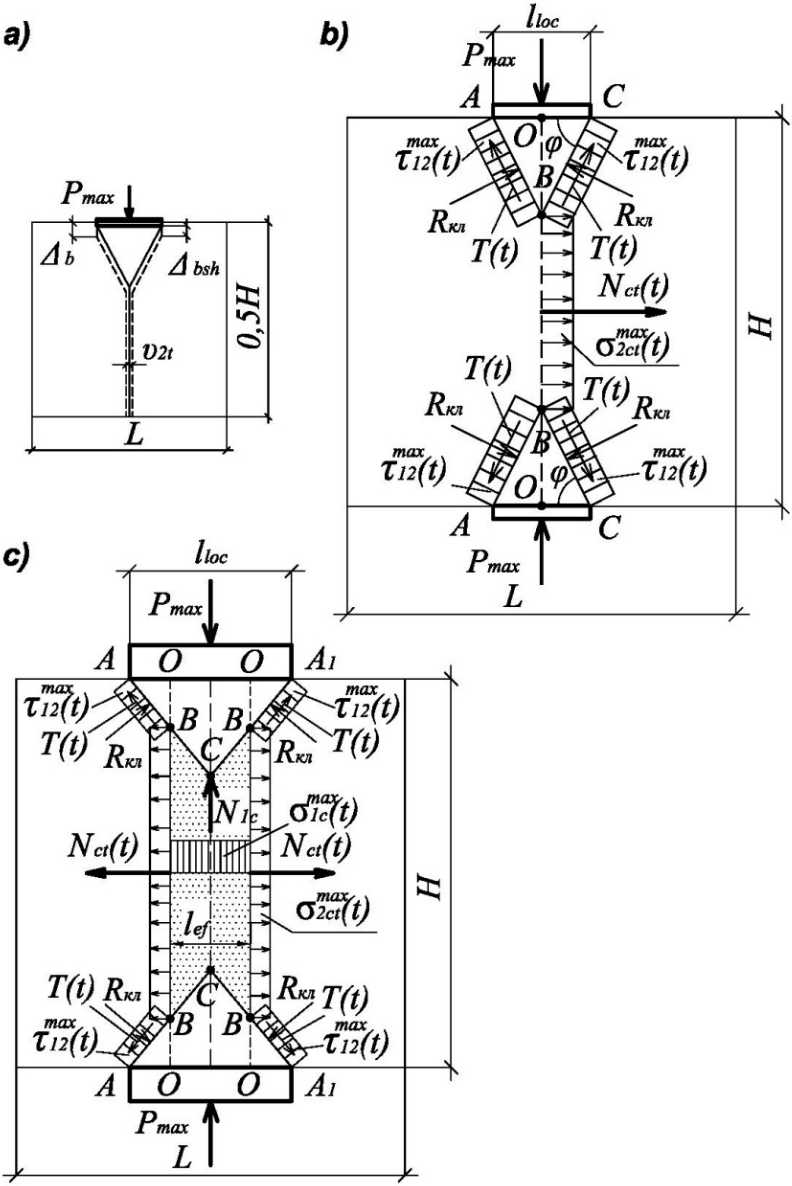

Fig. 3.1 Model of concrete deformation at local compression by repeated load (a), stress and forces distribution in compressed elements with zero shear span at cycle load at lloc / H < 0,2 (b) and lloc / H > 0,2 (c)

Thus, the criterion of fatigue destruction of concrete at local repeated compression can be shown as $\sigma_{1c}^{max}(t) > f_{cd,rep}(t)$, where $f_{cd,rep}(t)$ is the objective (residual) concrete strength in compression force flow of stresses at cycle load at the moment of time *t*; $\sigma_{1c}^{max}(t)$ is maximum compression stress of cycle from external load at the moment of time *t*.

So, as a result of wedges pressure on environing concrete stress state “compression-tension” between the vertexes of wedges appears, inside the wedge – “compression-compression”, along the wedge edges the condition of pure shear is realized. In this case, on the one hand, up to fatigue failure all components of the stressed state remain less than the estimated resistance of the concrete at a one-time static load, that is, $\tau_{12}^{max}(t) < f_{csh}$, and on the other hand, even at external fixed cycle load (P_{max}; $\rho = const$), stressed state in concrete and reinforced concrete elements is unstable (nonstationary), i.e. at repeated loading there is a continuous shift of stress-strain state of elements inside the compression force flow, formed between areas, through which, the load is transmitted. The reason for this is intensive extension of vibro-creep deformations $\varepsilon_{1c,pl}$ in compressed concrete along the axle of compression force flow.

Based on kinematic compression model of S.M. Krylov, L.N. Zaycev, I.S. Ulbiyeva [29], model of concrete deformation at local compression by repeated load can be shown Fig. 3.1, a, according to which, as a result of intensive extension of vibro-creep deformations of compressed concrete, there is a displacement of impact wedges in the vertical direction on value

$$\Delta_c = \int_{0,5 l_{loc}\cos\varphi\sin\varphi}^{0,5H} \varepsilon_{1c}(h)\,dh \qquad (3.1)$$

At vertical wedge displacement, along the axle of compression force flow on value Δ_c, transverse displacement in concrete in the middle zone will be

$$\upsilon_{2t} = \int_{0,5 l_{loc}\cos\varphi\sin\varphi}^{0,5H} \varepsilon_{1c}(h)\,dh \cdot \frac{1}{tg\varphi} \qquad (3.2)$$

And shear of concrete along the edges of impact wedge:

$$\Delta_{csh} = \int_{0,5l_{loc}\cos\varphi\sin\varphi}^{0,5H} \varepsilon_{1c}(h)dh \cdot \frac{1}{\sin\varphi} \tag{3.3}$$

Transverse displacement υ_{2t} calls causes uprising of additional tensile stresses $\sigma_{2ct}^{add}(t) = f_1(\upsilon_{2t})$ in concrete and shear, uprising of additional tangential stresses $\tau_{12}^{add}(t) = f_2(\Delta_{csh})$ in concrete along the edges of wedges. At increment of numbers *N* of cycle loads, vibro-creep deformations $\varepsilon_{1c,pl}$ of concrete increase. In turn, they cause enlargement of residual tensile stresses $\sigma^{add}(t)$ and residual tangential stresses $\tau_{12}^{add}(t)$ in concrete, which are distributed relatively evenly and have the same sign as initial stresses $\sigma_{2t}^{max}(t_0)$, $\tau_{12}^{max}(t_0)$ at first load, to maximum level of series load P_{max}. It is obvious that total stresses $\sigma_{2ct}^{max}(t) = \sigma_{2ct}^{max}(t_o) + \sigma_{2ct}^{add}(t)$ and $\tau_{12}^{max}(t) = \tau_{12}^{max}(t_0) + \tau_{12}^{add}(t)$. Stresses enlargement at repeated loads occurs not just at maximum value of external load, but at minimum, too. Thus, the actual factors of asymmetry of stresses series in concrete $\rho_{\sigma_{2ct}}$ and $\rho_{\tau_{12}}$ do not converge with the factor of asymmetry of external load series $\rho = P_{min} / P_{max}$. As the number of cycles and load levels increase, continuous increasing of maximum splitting normal $\sigma_{2ct}^{max}(t)$ and tangential $\tau_{12}^{max}(t)$ stresses in concrete and their factors of cycle asymmetry $\rho_{\sigma_{2ct}}$ and $\rho_{\tau_{12}}$ occur. Concerning this, based on the models of splitting of I.A. Rokhlin [30] and O.S. Zalesov, V.N. Sakharov, A.V. Starchevskiy [31] (3.1)…(3.3) Fig 3.1, *a*. the distribution of current stresses at repeated loads in concrete elements with dimensions $H \le 1{,}5L$ and $l_{loc} / H < 0{,}2$, can be represented as Fig. 3.1, *b*. Based on the compression model of B.S. Sokolov [28], the distribution of current stresses at repeated loads for concrete elements with dimensions $H \le 1{,}5L$ and $l_{loc} / H > 0{,}2$, are shown in Fig. 3.1, *c*.

Due to longitudinal fatigue micro cracks forming, following concrete resistance to destruction depends on concrete's ability to resist the extension of fatigue micro and macro cracks. For analytical description of fatigue destruction process and changing of fatigue strength of the concrete, it is expedient to use the destruction mechanics methods. Based on this, it is necessary to make up the equilibrium equation of forces at the moment of time *t* for evaluation of the objective (residual) strength at cycle loading, i.e. limit of durability of concrete element at local compression. For concrete element with dimensions $H \le 1{,}5L$ and $l_{loc} / H < 0{,}2$ (Fig. 3.1, *b*) – conditions of equilibrium of vertical and horizontal forces for half-wedge ABO, and the condition of horizontal forces equilibrium for vertical section OO; in elements with dimensions $H \le 1{,}5L$ and $l_{loc} / H > 0{,}2$ (Fig. 3.1, *c*). As a result of impact wedges pressure as a solid on concrete, that environs it, compression stresses are appearing $\sigma_{1c}^{max}(t)$ in it too. Consequently, in the core of force flow, there is compression force in concrete N_{1c}, which must be considered in stated conditions of equilibrium.

Due to uniform distribution of maximum actual stresses $\sigma_{2ct}^{max}(t)$, $\tau_{12}^{max}(t)$ and $\sigma_{1c}^{max}(t)$ in the process of cycle loading, and geometric dimensions of compression models after simple transformations, we get the analytic expression of the objective (residual) strength of concrete in compression force flow at cycle loading at the moment of time *t*, that is considering:

$$f_{cd,rep}(t) = \frac{k_{ccf}(t)}{\sqrt{\pi l(t,\tau)} \cdot Y(l)} \cdot \frac{\mathrm{h_t}}{l_{loc}} ctg\varphi \times$$

$$\times \left\langle A - B \cdot G_c \cdot L_\varepsilon \left\{ \frac{1}{E_c} + C_e \prod_{k=1}^{k=n} K_k \cdot a \cdot \psi_\upsilon + \int_{t_o}^{t} \frac{\partial}{\partial \tau} \left[\frac{1}{E_c(\tau)} + C_e(t,\tau) \right] dt \right\} \right\rangle^{-1} \tag{3.4}$$

where $A = 1$ and $B = 1/\sin^2\varphi$ – for concrete elements with dimensions of $H \le 1{,}5L$ and $l_{loc} / H < 0{,}2$, and for concrete elements with dimensions of $H > 1{,}5L$;

$A = \cos^2\varphi$ and $B = ctg^2\varphi$ – for concrete elements with dimensions of $H \le 1{,}5L$ and $l_{loc} / H > 0{,}2$;

h_t – tensioned zone length;

$$L_\varepsilon = \frac{1}{\pi}\left((2\theta_K - \pi) \cdot tg\theta_K - (2\theta_H - \pi) \cdot tg\theta_H\right)$$

де $\theta_H = arctg \sin\varphi \cos\varphi$, $\theta_K = arctg \dfrac{H}{l_{loc}}$.

As can be seen from Fig. 3.1, *c*, the limit of durability (objective fatigue strength) $f_{cd,\,rep}(t)$ of the concrete at local cycle compression, depends on critical factor of concrete stresses at repeated load $K_{ccf}(t)$ at the moment of time t, which is considered, on fatigue separation crack length $l(t)$ inside the compression force flow at the actual moment of time and deformation properties of concrete, and depends on the angle of internal concrete shear φ, elements dimension ratio H/L and load areas to element height ratio $\delta = l_{loc}/H$. Since the critical ratio of concrete stresses intensity at cycle load goes down, and the separation fatigue crack length inside the compression force flow increases and inelastic deformations of concrete with cycle numbers enlargement increase, the objective strength of the concrete has a variable value.

It is obvious that the presence of reinforcement affects the nature of extension of separation cracks, nature of compressed strip destruction and the value of objective strength $f_{cd,\,rep}(t)$ at local cycle compression load.

As a result of joint work of concrete and reinforcement repeated load, due to vibro-creep of compressed concrete, additional (residual) stresses appear and accumulate in the reinforcement. Accumulation of residual stresses in concrete and reinforcement that are equal by sign with initial stresses, leads to the increase of actual stresses in concrete and reinforcement and their ratios of cycle asymmetry. Even at continuous (stationary) external cycle load (P_{max}; $\rho = const$), with an increase of its cycle amounts, there is a continuous increase of maximum stresses $\sigma_{sc}^{\max}(t)$ and ratios of cycle asymmetry $\rho_{sc}(t)$ in vertical compressed reinforcement A_{sc}, maximum stresses $\sigma_{s}^{\max}(t)$ and ratios of cycle asymmetry $\rho_s(t)$ in horizontal reinforcement A_s. They are located in the middle zone between vertexes of impact wedges. Increase occurs in maximum tangential stresses $\tau_{12}^{\max}(t)$ and their cycle asymmetry ratios $\rho_{\tau_{12}}(t)$ in the concrete along the edges of impact wedges, and forces in horizontal reinforcement A_{sg}, that are crossed by shear area along the edges of impact volumes and maximum tangential stresses $\tau_{s}^{\max}(t)$ in them and asymmetry factors $\rho_{s\tau}(t)$.

The base of fatigue resistance of reinforced concrete element model at local compression, serves the model of fatigue resistance of the concrete element. Its geometric parameters and principles of formation converge. Stress distribution in concrete of reinforced concrete element as at first load, and at the process of cycle loading, is taken into account. Concrete deformations schemes are the same as in concrete elements (Fig. 3.1 *a*, *b*, *c*). External concrete resists to vertical displacement of impact wedges in reinforced concrete elements. The vertical displacement of the seal wedges in the reinforced concrete elements rests around the concrete, and the effect of the horizontal and vertical reinforcement in the above-mentioned equilibrium conditions for the semi-wedge ABO and for the vertical section of the OO (Fig. 3.1, b, c) is taken into account in the form of forces in the reinforcement

$$N_{sc}^{max}(t);\; Q_{s}^{max}(t);\; N_{s}^{max}(t).$$

Here from, we get the analytical expression of objective strength of reinforced concrete in compression force flow at cycle load at the moment of time *t*:

$$f_{cd,rep}(t)=\frac{h_t ctg\varphi}{l_{loc}\sqrt{\pi l(t)}\cdot Y(l)}\left\{k_{ccf}(t)+\sum_{i=1}^{h}\frac{\sigma_{si}^{max}(t)A_s}{b\cdot\sqrt{\pi\cdot l}(t)}\left\langle\sqrt{\frac{l(t)+(i-0,5)\cdot s}{l(t)-(i-0,5)s}}+\sqrt{\frac{l(t)-(i-0,5)\cdot s}{l(t)-(i+0,5)\cdot s}}\right\rangle\right\}\times$$

$$\times\left\langle A-\left\{G_c L_\varepsilon B+\frac{6E_s J_s L_\varepsilon\cdot n}{b\cdot\left(d_s\cdot\sqrt[4]{\frac{E_s}{E_c}}\cdot\left(1,4+1,25\sqrt[4]{\frac{a_s}{d_s}}\right)\right)^3}+C\right\}\right.$$

$$\left.\times\left\{\frac{1}{E_c}+C_e\prod_{k=1}^{k=n}K_k a\cdot\psi_\upsilon+\int_{t_o}^{t}\frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)}+C(t,\tau)\right]dt\right\}\right\rangle^{-1}$$

(3.5)

where *A*, *B* – see explanations (3.4); *C* = 0 – for reinforced concrete elements with dimensions $H \le 1,5L$ and $l_{loc} / H < 0,2$, and for reinforced concrete elements with dimensions $H \le 1,5L$ and $l_{loc} / H > 0,2$; $\sigma_{si}^{max}(t)$ and *s* – actual stresses in horizontal tensed reinforcement and its step.

For durability provision for concrete and reinforced concrete elements with zero shear span ($a_0 / h_0 = 0$), i.e. at local compression, it is necessary to determine the compression stresses $\sigma_{1c}^{max}(t)$, which appear inside the compression force flow, and limit them by concrete (or reinforced concrete) durability limit at local compression $f_{cd,rep}(t)$ for given mode of cycle load, i.e. inside the compression force flow, it is necessary to provide the condition of durability:

$$\sigma_{1c}^{max}(t)\le f_{cd,rep}(t) \qquad (3.6)$$

Since the extension of vibro-creep deformations $\varepsilon_{1c,\,pl}$ in compressed concrete in the direction of stress action $\sigma_{1c}^{max}(t_0)$ is going in free conditions and nothing interrupts their extension, we can accept $\sigma_{1c}^{\partial o \partial}(t) = 0$ and, thus, actual compression stresses $\sigma_{1c}^{max}(t)=\sigma_{1c}^{max}(t_0)$ at first load. Ratio of stress cycle asymmetry is equal to ratio of asymmetry of external load cycle, i.e. $\rho_c = \rho = P_{min} / P_{max}$. Limits of concrete durability are determined by (3.4) or (3.5).

3.2 Reinforced concrete elements resistance models with small, middle and large shear spans

Further researches of authors and I.T. Mirsayapov [27] and other have shown, that at $c_0 / h_0 > 2$ fatigue destruction of near support parts of bending elements occurs with the appearance of critical inclined crack, whose location is connected, not only with points of external force applying and support reaction, but with internal force factors, which occurs in shear span (moments and transverse forces). At $1,2 < c_0 / h_0 \le 2$ destruction of near support parts of beam elements at cycle load has little similar signs of destruction of elements with small and large shear spans. In this case, internal force factors and local concentrations of stresses in corresponding zones near points of external concentrated forces, influenced the behaviour of appearing and extension of cracks and fatigue destruction in this zone at indicated load.

The feature of “long” bending reinforced concrete elements work at small shear spans ($a_0 < 1,2h_0$) appears on local stress strips, connected with points of concentrated forces applying, within which fatigue destruction occurs. This feature of usual reinforced concrete beams with small shear spans joins them with “short” (high) elements. In both cases, this feature occurs at small values of relative distance between the forces, applied to the element.

T.I. Baranova, O.S. Zalesov [32], B.S. Sokolov [28] and other, consider that for practical calculations of “short” elements, the simplest solution of the problem is formation of calculating model as a frame-rod system (FRS), which consists of inclined compressed strips and tensed bottom, and compressed top reinforcing zones, which enclose at points of concentrated forces and support reactions applying (Fig. 3.2).

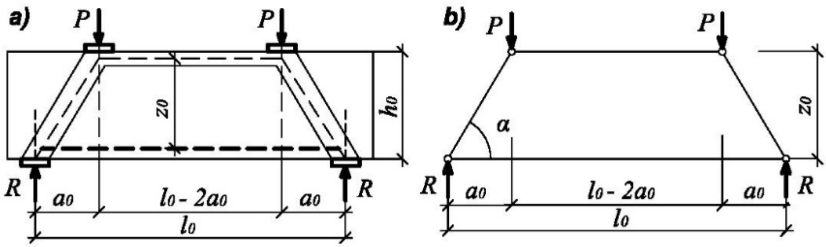

Fig. 3.2. Formation of force flows in usual ("long") beam with small shear spans at repeated load (a) and its frame-rod analogue (b)

Frame-rod analogue is widely used in the practice of reinforced concrete structures design abroad. In the last 30 years, in our country a lot of experimental and theoretic researches on the development of calculation models of short elements at static load in form of frame-rod system which confirmed its usability even at static load of experimental beams with shear span a_0 < 1,2h_0, have been performed.

The principle of calculation model development is in determining the compression stresses in inclined force flows and tensile stresses in horizontal flow, intersection of which creates a system which can, conditionally, be called as frame-rod model of short elements. Main parameters which determine calculation inclined strips are dimensions of load l_{sup}^{top} and support l_{loc}^{bot} areas, under which flows of compression stresses are forming. The smaller the sizes of areas, the highest trajectory density. So, support and load areas form an incline strip and its width at top and at bottom. The angle of incline of main compression stresses flow, approaches the angle of line incline which connects centres of applying of support reaction and external concentrated force.

It is obvious that in the process of modelling of near support area of the concrete, element works at small shear spans using frame-rod analogue and it could be considered that its fatigue strength is determined by durability of every FRS element: inclined compressed strips and strength of tensed reinforcement. Fatigue destruction of tensed elements zone occurs as a result of fatigue rupture of longitudinal reinforcement at place of intersection with inclined crack, or as a result of violation of reinforcement anchoring by inclined crack. Thus, occurring stresses should be limited by values of objective concrete and reinforcement strength at cycle load (durability limit) and its friction between themselves, i.e. for provision of durability of such reinforced concrete elements, it is necessary to follow the durability conditions:

$$\sigma_{1c}^{\max}(t) \le f_{cd,rep}(t),\ \sigma_{s,в}^{\max}(t) \le f_{ydq,rep}(t)$$
$$\sigma_{s}^{\max}(t) \le f_{yd,an}(t) \qquad (3.7)$$

where $\sigma_{1c}^{\max}(t)$ – compression stress on compressed force flow;

$\sigma_{s,в}^{\max}(t)$ – actual tensile stresses in most loaded fibres of longitudinal reinforcement in the place of intersection with inclined crack;

$\sigma_{s}^{\max}(t)$ – actual (maximum) axle tensile stresses in longitudinal reinforcement in the place of intersection with inclined crack;

$f_{cd,rep}(t)$ – limit of concrete durability at local compression;

$f_{ydq,rep}(t)$ – limit of reinforcement durability on tension;

$f_{yd,an}(t)$ – limit of longitudinal reinforcement anchoring durability.

Experimental researches [27, 28, 32] have shown, that stress-strain state inside the inclined compression force flow, is the same as in flat-stressed elements at local load. Thus, for evaluation of fatigue strength of inclined strip, the model of fatigue destruction at compression and equations of objective (residual) strength of the concrete and reinforced concrete at cycle load, can be used. Wherein, if axle "1" (Fig. 3.3) is directed along the longitudinal axle of inclined compression force flow, and axle "2" in orthogonal direction, and use the same designations as in elements with zero shear span, stress state inside the inclined compressed force flow can be represented as Fig. 3.3.

Since the vibro-creep deformations $\varepsilon_{1c,\,pl}$ extension in compressed concrete in the direction of stresses $\sigma_{1c}^{\max}(t_0)$ action, as at local compression, occurs in free conditions and nothing interrupts its extension, we can accept that $\sigma_{1c}^{add}(t) = 0;\ \sigma_{s}^{add}(t) \approx 0;\ \sigma_{1c}^{\max}(t) = \sigma_{1c}^{\max}(t_0);\ \sigma_{s}^{\max}(t) \approx \sigma_{s}^{\max}(t_0),$ and i $\sigma_{s}^{\max}(t_0)$ slightly simple determines at first load from the conditions of equilibrium on the base of fatigue resistance model.

Because stress-strain state inside the inclined compressed strip and the behaviour of its fatigue destruction are analogical to stress-strain state and the behaviour of fatigue destruction of flat-stressed elements at local load action, expression for determining the objective fatigue strength (limit of durability) of inclined compressed stripe at the moment of time *t* by analogy (3.4) and (3.5) takes the form:

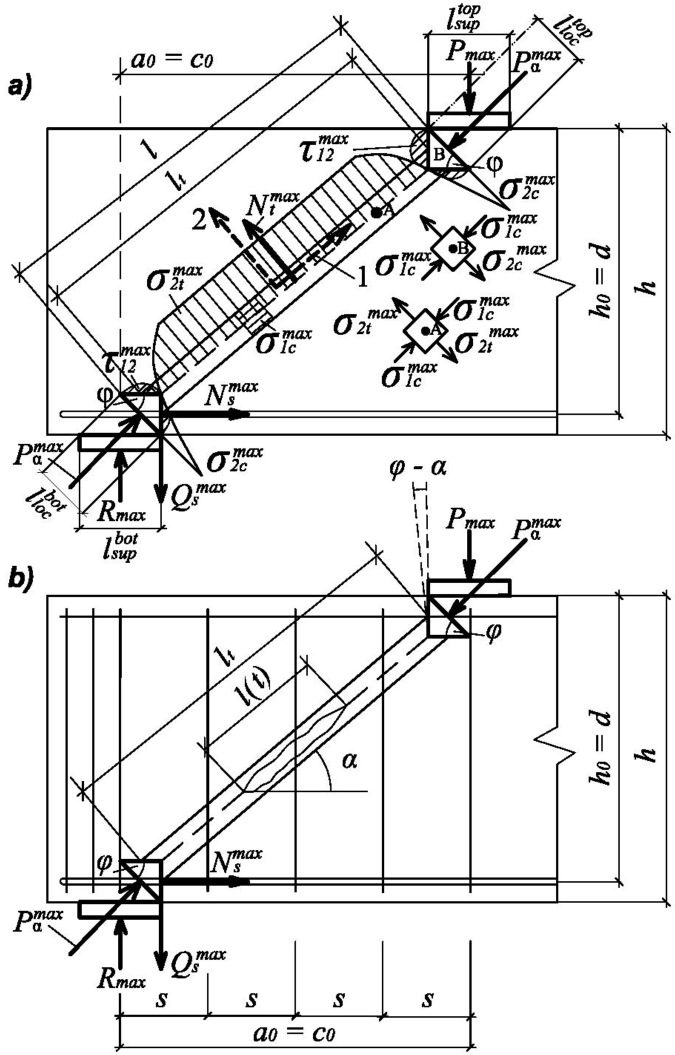

Fig. 3.3. Physic model (a) and calculation scheme (b) of bending reinforced concrete element resistance with small shear span at joint action of transverse force and bending moment

$$f_{cd,rep}(t)=\frac{\left(k_{ccf}(t)+K_{isw}(t)\cos\alpha\right)\cdot l_t\, ctg\varphi}{l_{\sup}\sin\alpha\sqrt{\pi l(t)}\,Y(l)}\times$$

$$\times\left\langle A_{нп}-\left\{G_c L_\varepsilon B_{нп}+\frac{6E_s I_s L_\varepsilon\cdot n\cdot\cos(\varphi-\alpha)\sin\alpha}{\mathrm{b}\left(d_s\sqrt[4]{\frac{E_s}{E_c}}\left(1{,}4+1{,}25\sqrt[4]{\frac{a_s}{d_s}}\right)\right)^3\sin\varphi}\right\}\times\right.$$

$$\left.\times\left\{\frac{1}{E_c}+C_e\prod_{k=1}^{k=n}K_k a\psi_\upsilon+\int_{t_o}^{t}\frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)}+C(t,\tau)\right]dt\right\}\right\rangle^{-1} \quad (3.8)$$

where $K_{isw}(t)$ – stress intensity factor which characterizes influence of transverse reinforcement on crack extension inside incline compressed flow;

α – angle of compressed strip incline;

$A_{нп}$ = 1, $B_{нп}=1/\sin^2\varphi$ – for reinforced concrete elements with load areas sizes l_{sup} / h < 0,2; $Aнп = cos^2\varphi$, $Bнп = ctg^2\varphi$ – for reinforced concrete elements with load areas sizes l_{sup} / h ≥ 0,2; in elements without transverse reinforcement K_{isw} = 0.

Multi-cycle fatigue of reinforcement is characterized by appearance and extension of fatigue cracks in it. The formation of fatigue cracks occurs as a result of intensive plastic deformation of reinforcement steel in local volumes of stress concentration in reinforcement, main source of which, is its periodic shape. It leads to significant closed hysteresis loops, whose area is equal to energy spent by one cycle of the load. After plastic deformations are exhausted, micro cracks appear in these local volumes, one of which can transform into the main crack. Following the enlargement of cycle numbers, extension of the main crack up to the critical size, occurs. Thus, for analytical description of fatigue destruction process and change of fatigue strength of steel reinforcement in reinforced concrete element at repeated loads, methods of destruction mechanics are used. The durability limit (objective strength of longitudinal reinforcement at the moment of time *t*, at the place of its intersection with inclined crack in conditions of flat stressed state becomes:

$$f_{sd,в}(t)=\sigma_{sc}\cdot K_{scf}(t)\Big/\sqrt{(Y(l)\cdot\sigma_{sc})^2\cdot l_s(t)+K_{cf}^2(t)} \quad (3.9)$$

$$\sigma_{sc}=\sigma_u\Big/\left\langle 1+\exp\left(-2E_s\cdot\varepsilon_{pl}^{pec}\big/\sigma_{su}\right)\sqrt{1+3\left(\tau_{si}^{max}\big/\sigma_{sвi}^{max}\right)^2}\right\rangle \quad (3.10)$$

where $\sigma_{sb,\max}$, $\tau_{s,\max}$ – normal stresses in most loaded (tensioned) fibres and tangential stresses in longitudinal reinforcement in the place of its intersection with inclined crack;

$l_s(t)$ –length of fatigue crack in reinforcement at the moment of time *t*;

K_{scf} – critical factor of reinforcement stresses intensity at repeated loads at the moment of time *t*;

σ_{su} – temporary steel resistance to rupture;

ε_{pl}^{pec} – residual plastic resource of steel.

Process of multi-cycle fatigue anchoring of reinforcement is characterized by the appearance and extension of fatigue cracks in the contact zone between the reinforcement and concrete. If the clutch stresses of reinforcement and concrete τ_g are high, and these stresses are larger than limit of durability of the clutch, i.e. condition τ_g / τ_{rep} > 1 is true, generation and extension of through (inner) fatigue cracks in the contact zone between the reinforcement and concrete occur. As it is shown in researches by B.Broms, I. Goto [33], M.M. Kholmyanskiy [34], M.I. Karpenko [18] these through inner cracks form cone-shaped volumes. Indicated cracks permeate into concrete thickness, which crumples under these protrusions. Thus, objective fatigue strength of the concrete under protrusions and forces of clutch of reinforcement protrusions with concrete, should be determined as function of cone-shaped crack length *l*(*t*), which is permanently increasing with increasing of load cycle number. However, for analytical characteristic of process of fatigue destruction of contact zone, and for change of fatigue strength of longitudinal reinforcement anchoring at repeated loads, it is also expedient to use methods of destruction mechanics. Then, the limit of durability (objective strength) of longitudinal reinforcement anchoring at the moment of time *t* is determined by:

$$f_{yd\ \ an,rep(t)} = k_{ccf}(t)ctg\varphi\left(\frac{1{,}5a}{\cos\varphi_k} - \frac{c_r}{\varphi_k}\sin\varphi\cos\varphi\right)\times$$
$$\times\left(d + 2c_r + \left(0{,}75a - 0{,}5c_r ctg\varphi_k \sin\varphi\cos\varphi\right)\right)\times$$
$$\times\left(1{,}5\left(1+\sin\alpha_r\right) - \sqrt{\sin\alpha_r}\right)\cdot\frac{2\tau_g\left(d+2c_r\right)\left(L+L_{pl}\right)}{d^2}\times \tag{3.11}$$

$$\times\left(\sqrt{\pi l(t,\tau)}\cdot Y(l)s_r(d+2c_r)\sin 2\varphi_k \sin\alpha_r\right)^{-1}\times$$
$$\times\left\langle 1-\frac{G_c(3atg\varphi_k-2c_r\sin\varphi\cos\varphi)}{c_r\cos\varphi\sin^2\varphi}\frac{A_{sh}}{A_c}\left\{\frac{1}{E_c}+C_e\prod_{k=1}^{k=n}K_k a\psi_\upsilon+\int_{t_0}^{t}\frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)}+C(t,\tau)\right]dt\right\}\right\rangle^{-1}$$

where $\frac{A_{sh}}{A_c}=\frac{0,5\cos\varphi}{(d+c_r)}\left\{d+2c_r+\frac{0,5c_r\sin(\varphi-\varphi_k)}{\sin\varphi_k\cos\varphi}\right\}$;

d - rod diameter;

c_r, s_r, α_r – accordingly height, step and angle of incline of reinforcement protrusions;

a – concrete cover;

L, L_{pl} – the length of reinforcement fastening in concrete and plastic place of this fastening;

φ_k – angle of wedge under reinforcement protrusions;

$l(t,\tau)$ – length of fatigue crack in concrete under reinforcement protrusions at the moment of time t.

During cycle loading under the influence of high stresses of concrete crumpling under the reinforcement protrusions, there are intensively extents deformations of vibro-creep. With enlargement of load cycles number *N*, due to concrete vibro-creep under reinforcement protrusions which surrounds them, increasing of displacement increment $g_0^{max}(t)$ on loaded end and inside the fastening $g_x^{\max}(t)$ occurs, and this, in its turn, leads to redistribution of clutch forces $P_{i,r}$ from more loaded protrusions in the end of fastening, to protrusions that are situated in the depth of fastening, i.e. redistribution of clutch stresses τ_g along fastening occurs. Wherein, enlargement of load cycle number leads to continuous increasing of plastic area length and increasing of completeness of clutch stress diagram.

Analysis of a number of experimental data shows that fatigue strength and limit of durability of reinforced concrete bending elements in the zone of joint action of transverse forces and bending moments, exceeds appropriate stresses (loads), at which, inclined cracks in the tensed zone of the element appear, even for short time static load, i.e. bending reinforced concrete structures resist to repeated cycle loads at presence of normal and inclined cracks in near support areas. Concerning this, during the development of calculation model for evaluation of fatigue strength or durability of such structures at the transverse force and bending moment action, it is necessary to consider the existence of cracks in the tensed zone, because the appearance and the extension of inclined cracks, radically change the quality of stress-strain state, especially, in the elements with large shear spans.

The condition of the formation of cracks in the stretched zone of elements under the vertical displacement of wedges in sealing concrete elements that resists surrounding concrete, and the impact of horizontal and vertical reinforcement in the above-mentioned equilibrium conditions for the semi-wedge ABO and for the vertical section of the OO (Fig. 3.1, b, c) is taken into account in the form of forces in the reinforcement.

In elements with large shear spans ($a_0 / h_0 > 2$) in the zone of joint action of transverse forces and bending moments, first of all normal cracks appear, and then, at optimal quantity of longitudinal work reinforcement (in not over reinforced structures), they are warping on near support areas by trajectories of main compression stresses and transform into inclined cracks. By increasing the cycle number, one of such inclined cracks starts to expend more intensively and becomes critical. Trajectory of main compression stresses, along which appearance and extension of initial place of critical incline crack occur, can be described by equation $y/h = m/(n + h/a)$, where m; n – are determined from boundary conditions. Analysis of character of appearance and expansion of fatigue cracks, fatigue destruction of experimental beams, their stress-strain state in the zone of joint action of transverse forces and bending moments at repeated loads of high level and experimental thermograms [27] of near support areas of experimental elements, allow to propose the following hypothesis of following expansion of critical inclined crack and developing the physic model of fatigue destruction of bending reinforced concrete elements with large shear spans. Long before the appearance of normal and inclined cracks in shear span, especially, before forming end expansion of critical inclined crack, in normal section, at the end of shear span, where maximum moment occurs, normal crack appears (section 1-1 on fig. 3.4).

Until the remaining cracks appear in the zone of transverse force and bending moment action, this normal crack in the end of shear span extends on high

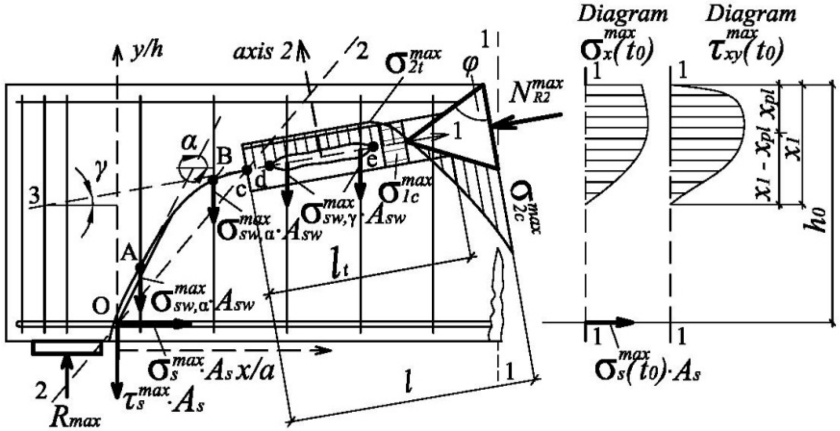

Fig. 3.4 Physic and calculation model of fatigue resistance of not over reinforced concrete element with large shear span

height and tensed zone is practically fully off from work, the diagram $\sigma_x^{max}(t)$ is twisted, increases the completeness of this diagram ω_σ and in top part of it, plastic area starts to form; reduction of compressed part of concrete section height which has no cracks, yet leads to sharp increase of completeness of diagram ω_τ of tangential stresses and to sharp increase of maximum value of tangential stresses $\tau_{xy}^{max}(t)$. Thus, inside the plastic area x_{pl} of compressed zone, it is sharply increased resulting N_{R2}^{max} of normal

$$N_c^{max} = \int\limits_{A_{pl}} \sigma_x^{max}(t) \cdot dA$$ and tangential $$Q_c^{max} = V_c^{max} = \int\limits_{A_{pl}} \tau_{xy}^{max}(t) \cdot dA$$

forces, where A_{pl} – area of plastic part of compressed zone in normal section with crack at the end of shear span. Under the influence of the force N_{R2}^{max} in a compressed zone operating in limits of load area $x_{pl}/\cos\gamma$, in the direction of the line of this force there is an inclined compressive force stream inclined at an angle γ to the longitudinal axis of the element. Pattern of stress distribution inside this inclined compressed force flow, is the same as at local compression. Action line of tensile stresses generates and extend fatigue separation microcracks during the cyclic loading.

That happends before the appearance of critical inclined crack inside the inclined compressed force flow from micropores in concrete body and extend fatigue separation microcracks, that later join into separation macrocrack *ed* at angle *γ* to longitudinal element axle.

The most characteristic feature of normal separation cracks on near support parts of not over reinforced beams, is the tendency of any, even initially inclined to compression force action line, crack to align its trajectory in the direction of this force. Wherefrom, we can accept the hypothesis, that from all inclined cracks which were formed on near support part from joint action of transverse force and bending moment in the tensed zone at first load or at increasing of cycle numbers and load levels, that inclined crack becomes critical, which comes to zone of influence of inclined compressed force flow, generated by action of resulting N_R^{max} of forces in compressed zone inside the plastic area x_{pl}. This can only be explained by the fact that the critical, as a rule, becomes the extreme (closest to the support) fracture that forms and develops along the less stressed trajectory of the main compressive stresses. What is happening further is development of the critical sloping crack and its disclosure that is more intensive in comparison with other inclined cracks. Also, there is a severe increase in normal stresses in the longitudinal reinforcement in place of its intersection with a critical sloping crack, that is, the alignment of longitudinal forces.

Fatigue destruction of reinforced concrete element on the inclined section occurs due to compressed zone or as a result of fatigue rupture of the most stressed rods of transverse reinforcement which intersects with initial area of critical inclined crack, or on the tensioned zone as a result of fatigue rupture of longitudinal reinforcement in normal section 1-1, or because of anchoring violation of longitudinal reinforcement on and out of support.

So, for assurance of operability of the element at repeated load, one needs to adhere to conditions:

$$\sigma_{1c}^{max}(t) \le f_{cd,rep}(t),\ \sigma_{sw,\alpha}^{\max}(t) \le f_{ydw,rep}(t),$$
$$\sigma_{s\text{в}}^{\max}(t) \le f_{ydq,rep}(t),\ \sigma_{s}^{\max}(t) \le f_{ydan,rep}(t) \qquad (3.12)$$

where $\sigma_{sw,\alpha}^{\max}(t)$ − actual maximum stresses in the most loaded rods of transverse reinforcement at the moment of time *t*, on the place of their intersection with initial part of critical inclined crack in the tensed zone;

$f_{ydw,rep}(t)$ − durability limit of transverse reinforcement rods at their axle, loading at the moment of time *t*;

The same as in elements with large shear span, action of repeated load, which leads to extension of vibro-creep deformations of compressed concrete in the direction of stresses σ_{x1}^{max}, σ_{1c}^{max}, is accompanied by the appearance and extension of additional (residual) stress-strain state on near support part of bended reinforced concrete element. With the aim of stress-strain state evaluation simplification, the action of repeated load and reinforced concrete element work, is divided into two stages. The first stage shows the stressed state of the structure at the first cycle (N = 1) of load, to maximum cycle load P_{max}. The second stage is characterized by the stressed state of the element in the process of its repeated load ($N > 1$), which is continuously changed through intensive extension of vibro-creep deformations $\varepsilon_{1c,\,pl}$ of compressed concrete.

In the general flow, stresses in concrete and reinforcement, and the factors of cycle asymmetry, have the form:

$$\sigma_i^{\max}(t) = \sigma_i^{\max}(t_0) \pm \sigma_i^{add}(t) \qquad (3.13)$$

$$\rho_i(t) = \left\langle \rho \cdot \sigma_i^{\max}(t_0) + \sigma_i^{add}(t) \right\rangle \Big/ \left\langle \sigma_i^{\max}(t_0) + \sigma_i^{add}(t) \right\rangle \qquad (3.14)$$

where $\rho = \mathrm{P}_{\min} / P_{\max}$, $\sigma_i^{\max}(t_0)$ − initial stresses in concrete or reinforcement at the first half-cycle of load; $\sigma_i^{\partial o\partial}(t)$ − additional (residual) stresses in concrete or reinforcement, which appear as a result of concrete vibro-creep deformations accumulation.

Initial stresses at first load $\sigma_i^{\max}(t_0)$ are determined from conditions of external and internal forces equilibrium based on the model of fatigue resistance of the element. Additional stresses $\sigma_i^{add}(t)$, which appear during the process of their repeated load, starting from the second load cycle, are determined based on the deformation relation for normal section (1-1) at the end of shear span and inclined section (2-2), which are placed on the critical inclined crack (Fig. 3.4).

Fatigue destruction of compressed concrete zone over critical inclined crack, occurs under the action of resulting $N_{R2}^{\max}$ of transverse and longitudinal forces, which appear inside the plastic part of normal section 1-1. Due to stress-strain state of the compressed concrete zone over critical inclined crack (inside the inclined compressed force flow), and the behaviour of fatigue destruction, we can conclude that they are analogical to stress-strain state and the behaviour of fatigue destruction in flat-stressed elements at the local load action, objective fatigue strength over critical inclined crack at the moment of time *t*, so we determine:

$$f_{cd,rep}(t) = \frac{\left(k_{ccf}(t) + k_{1sw}(t)\right) \cdot l_t \cos\gamma ctg\varphi}{x_{pl}\sqrt{\pi l(t)} \cdot Y(l)} \times$$
$$\times \left\langle 1 - \left[\frac{G_c L_\varepsilon}{\sin^2\varphi} + \frac{6E_s I_s L_\varepsilon \cdot n \cdot \cos(\varphi - \gamma)\sin\gamma}{\left(d_s \cdot \sqrt[4]{\frac{E_s}{Ec}} \cdot \left(1{,}4 + 1{,}25\sqrt[4]{\frac{a_s}{d_s}}\right)\right)^3 \sin\varphi} \right] \times \right.$$
$$\left. \times \left\{ \frac{1}{E_c} + C_e \prod_{k=1}^{k=n} K_k a\psi_v + \int_{t_o}^{t} H_\sigma \frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)} + C(t,\tau)\right] dt \right\} \right\rangle^{-1} \qquad (3.15)$$

Durability limit of longitudinal reinforcement $f_{sd,b}(t)$ on the place of its intersection with critical inclined crack in conditions of flat-stressed state, is determined by (3.9) and (3.10). Durability limit of longitudinal reinforcement anchoring $f_{yd,an}(t)$ by critical inclined crack, is determined by (3.11). Durability limit $f_{ydw,rep}(t)$ at axial load, is determined by (3.9) and (3.10), taking into account that, $\tau_{sw}^{\max} = 0$.

Testing [27] reinforced concrete beams with rectangle cross section with shear span $a_0 = c_0 = (1{,}51 - 1{,}67)h_0$,has allowed us to specify the following picture of appearance and extension of cracks and character of fatigue destruction in the zone of transverse forces and bending moments action. Since the elements with middle shear span 1,2 $h_0 < c_0 = a_0 <$ 2 h_0 are on the borders of elements with small and large shear spans, in their operating and in mechanics of fatigue destruction at middle shear spans, features of the first and the second are determined, i.e. the behaviour of appearance and extension of cracks in the zone of transverse force and bending moment action and fatigue destruction of these elements is influenced by internal force factors. Local fields of stress state and stresses concentration in corresponding zones in places of concentrated external forces are applied. Thus, at the middle shear spans, the fatigue destruction occurs with the appearance of critical inclined crack, but local fields of stressed state and concentration of stresses in indicated zones influence the destruction. Critical inclined crack can appear on the distance (0,2…0,3) *h* from the tensed edge and extends in support or concentrated external force direction. In the tensed zone, it disclosures along the line 2-2 (Fig. 3.5), which connects the inner edge of supporting plate with the external edge of load plate, and fully intersects (to the inner edge of the support plate). But, in its extension from support to concentrated force, critical inclined crack, after approaching point *O,* i.e. intersection of lines 2-2 and 3-3, changes its direction and resumes extension along the line 3-3 by the axle of inclined compressed flow. At the same time, inside the compressed force flow on the line of tensile stresses $\sigma_{2t}^{\max}$, rupture crack $d - e$ along the axle 3-3 appears and extents, which afterwards merges with the initial part OO_2 of the critical crack. It is obvious that the appearance, extension and disclosure of the critical crack in tensed zone (area OO_2) are connected with flat rotation and shear of inclined section 2-2. Their extension and disclosure in compressed zone (*ed*) are caused by the appearance and the extension of micro cracks rupture on the line of tensile stresses $\sigma_{2t}^{\max}$ (Fig. 3.5) action in the zone of “tension-compression”, inside the inclined compressed force flow, formed under the action of force $P_{\beta}^{\max}$, and following

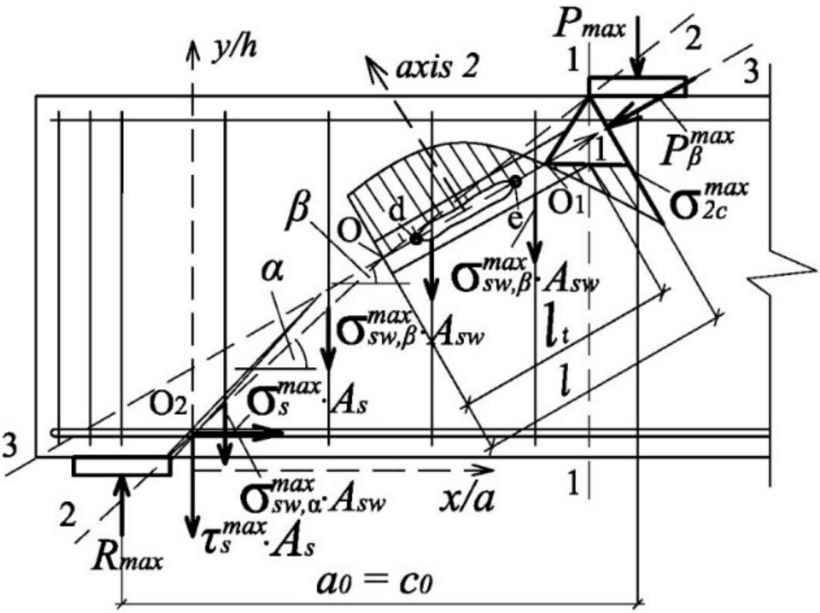

Fig. 3.5 Physic model and calculation scheme of fatigue resistance of inclined, not over reinforced element with middle shear span

merge into macro crack, following the extension and disclosure of this macro crack rupture. Behaviour of stress distribution inside the inclined compressed force flow is the same as at crumpling.

For this cause of stress-strain state and destruction character objective fatigue strength (durability limit) of inclined concrete strip over critical inclined crack becomes:

$$f_{cd,rep}(t)=\frac{\left[k_{ccf}(t)+k_{1sw}(t)\right]l_t ctg\varphi}{l_{sup}sin\beta\sqrt{\pi l(t)}Y(l)}\times \tag{3.16}$$

$$\times\left\langle 1-\left\{\frac{G_c L_\varepsilon}{sin^2\varphi}+\frac{6E_s I_s L_\varepsilon n cos(\varphi-\beta)sin\beta}{\left[d_s\sqrt[4]{\frac{E_s}{E_c}\left(1,4+1,25\sqrt[4]{\frac{a_s}{d_s}}\right)}\right]^3 sin\varphi}\right\}\times\left\{\frac{1}{E_c}+c_e\prod_{k=1}^{k=n}K_k a\psi_v+\int_{t_0}^{t}\frac{\partial}{\partial\tau}\left[\frac{1}{E_c(\tau)}+C(t,\tau)\right]dt\right\}\right\rangle^{-1}.$$

Durability limits of transverse and longitudinal reinforcement, and the durability limit of its anchoring are determined by (3.9), (3.10) i (3.11).

3.3 Conclusions on chapter 3:

1. The analysis of present methods of calculation of reinforced concrete structures at joint action of transverse forces and bending moments, shows that, in most cases, they are performing in assumption of elastic concrete work without considering its physic nonlinearity and change of deformation modes of materials in structures at cycle loading;

2. Considered physical models and calculation schemes of near support areas resistance of not over reinforced span, reinforced concrete structures, to repeated load of high level, different types of fatigue destruction of materials are envisaged considering vibrocreep deformations, accumulation of the damages in form of fatigue micro- and macro cracks.

CHAPTER 4 ENGINEERING METHOD OF INCLINED SECTIONS OF BEAM STRUCTURES CALCULATION ON THE FATIGUE FRACTURE MODEL

Practical application of calculation methods of used materials and reinforced concrete elements on durability by models reviewed in p. 3, is bound with some computational difficulties. Besides, special knowledge connected with the crack theory and destruction mechanics, that caused additional difficulties for the designers, are needed. At that, main difficulties occur evaluating the objective (residual) strength of concrete and reinforcement at repeated loads. They are related to accumulation of damages and reduction of plastic resource of material through integral parameters: fatigue crack length $l(t)$ and critical values of stress intensity factor at fatigue destruction $K_{cf}(t)$. In addition, the development of fatigue cracks in concrete, leads to increase of vibrocreep deformations, because concrete deformations at cycle load consist of linear and nonlinear parts. Nonlinear part of vibrocreep deformations appears as a result of creation and development of fatigue micro cracks in concrete, and is a function of fatigue macro crack length $l(t)$. In this connection, the determination of the nonlinear part of the deformation of creep vibration (vibrocreep) also encounters the corresponding difficulties. Determining these three parameters complicates the calculated dependencies. Linear part of vibrocreep deformation also meets appropriate difficulties. Thus, for practical calculations, we consider engineering method which is based on theoretical results of p. 3 and some simplifying preconditions. Change of the stress state in the process, as well as change of strength properties of the reinforcement, concrete and their adhesion, should be taken into account. Wherein, vibrocreep deformations, concrete and reinforcement fatigue strengths, reinforcement anchorage and stresses in them, are calculated by simplified method at the moment of time t.

The main element of the simplified technique is the application of simplified method for calculating the limits of concrete and reinforcement durability.

It is known that the change of the materials strength under cyclic loading in half-logarithmic coordinates $f_{cd} = lgN$ is described by linear dependence. The durability line is characterized by inclined and horizontal areas. Therefore, the characteristic points of the durability line are the beginning and the point of its inflection.

For concrete, the beginning of the durability line is a point on the axis of stress (Fig. 4.1) at *N* = 1, which corresponds to its dynamic strength at a one-time load at a speed equal to the speed of cyclic load applied. Herewith, the larger their frequencies, the greater the strength at a one-time dynamic load.

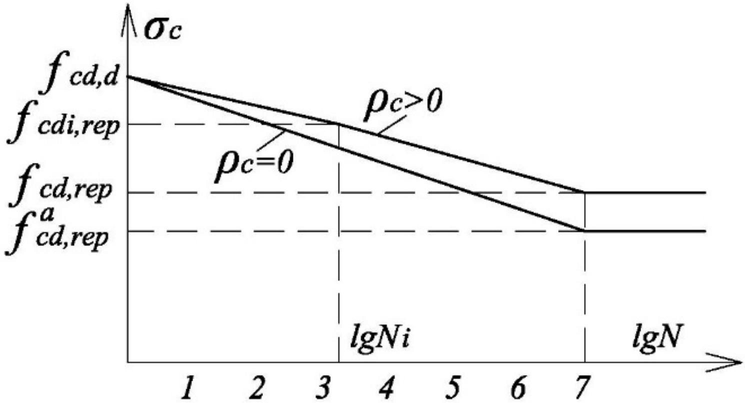

Fig. 4.1 Calculated durability concrete lines

In practical calculations, we accept:

$$f_{cd,d} = k_d \cdot f_{cd} \tag{4.1}$$

where k_d, f_{cd} – coefficient of dynamic strengthening of concrete and its strength at static loading.

According to the proposition [35], the coefficient of dynamic strengthening of the concrete is determined as loaded for the first time by the formula:

$$k_d = 1 + 0,6 \cdot \frac{0,27 + 0,8\left[th(0,15lg\upsilon) - 0,358\right]}{1 - 0,358tg(lg\upsilon)} \tag{4.2}$$

and taking into account the plastic resource at the previous stages of loading:

$$k_d = 1 + 0,085lg\upsilon \left\langle lg \left\{ \begin{array}{l} c_\infty(t,\tau) - \dfrac{c_\infty(t,\tau)\sigma_{c1}^{\max}(1-\rho_{c1})\left[1-(1-a)^N\right]10^5}{f_{cd}} + \\ + \dfrac{\sum_{i=2}^{k} \Delta\sigma_c c_\infty(t,\tau)(1-\rho_{b1})\left[1-(1-a)^N\right]10^5}{f_{cd}} \end{array} \right\} \right\rangle \tag{4.3}$$

where *v* – loading speed.

For concrete, the absolute durability limit at $\rho_c = 0$ at [35] varies within $(0,47\ldots0,55)f_{cd}$, and coincides with the lower limit of micro cracking. Therefore, in practical calculations, the absolute limit of durability $f^{a}_{cd,rep}$ at $\rho_c = 0$ equals $0,5f_{cd}$, and the relative durability limit in this case is:

$$k^{a}_{c,rep} = f^{a}_{cd,rep} / f_{cd} = 0,5 \tag{4.4}$$

The relative value of concrete durability at tensile, shearing and torsion are taken the same. Since the lower limit of micro cracking f^{0}_{crc} depends only on the level of the action stress and the type of stress state, then for any ρ_c , the durability limit will be the same, and its value will have an effect on the structure durability, that is, on the cycles number at which the durability lines will come to an end. As known, the overlap of these lines begins at $N \geq 10^7$. For $\rho_c = 0$, it can be assumed $N = 10^7$, which creates some reserve. For larger values of ρ_c this point moves to the right along the lgN axis, the greater, the larger ρ_c [35]:

$$k_{c,rep} = \frac{f_{cd,rep}}{f_{cd}} = \frac{k^{a}_{c,rep}}{1 - \rho_c\left(1 - \frac{k^{a}_{c,rep}}{k_d}\right)} \tag{4.5}$$

and durability strength at $N < 10^7$

$$f_{cdi,rep} = f_{cd,d} - \frac{lgN_i}{7}\left(f_{cd,d} - f_{cd,rep}\right) \tag{4.6}$$

or in relative values taking into account (4.1) i (4.6)

$$k_{ci,rep} = 1,3 - \frac{lgN_i}{7}\left(1,3 - \frac{0,5}{1 - 0,616\rho_c}\right) \tag{4.7}$$

Taking $k_d = 1,3$ and considering (4.4) we get:

$$k_{ci,rep} = 1,3 - \frac{lgN_i}{7}\left(1,3 - \frac{0,5}{1 - 0,616\rho_c}\right) \tag{4.8}$$

For reinforcement, A.P. Kirillov [35] recommends to accept the following expression as the beginning of the durability line:

$$f_{yd,d} = \eta \cdot \sigma_u \tag{4.9}$$

and the overlap of the durability line (Fig. 4.2) in the point with coordinates

$$f_{yd,rep} = \sigma_u \frac{k_0 \cdot k_c \cdot k_r}{1 - \rho_s\left(1 - \frac{k_0 \cdot k_c \cdot k_r}{\eta}\right)} \qquad lgN = 6,3 \tag{4.10}$$

where $k_0 = \dfrac{f_{yd0,rep}}{\sigma_u}$ – relative reinforcement durability limit at ρ_s = 0;

k_c – a coefficient that takes into account the presence of a weld joint or other stress concentrator;

k_r – a coefficient that takes into account the reinforcement diameter;

σ_u – temporary reinforcement resistance to breakage;

η = 1,8 – empirical coefficient.

The element fatigue strength for the number of cycles $N < 2{\cdot}10^6$ is:

$$f_{ydi,rep} = f_{yd,d} - \frac{lgN_i}{lg(2\cdot 10^6)}\left(f_{yd,d} - f_{yd,rep}\right) \tag{4.11}$$

and taking into account (4.9) and (4.10) in relative values, it has the form:

$$k_{ydi,rep} = \eta - \frac{\lg N_i}{6,3}\left[\eta - \frac{k_0 \cdot k_c \cdot k_r}{1-\rho_s\left(1-\dfrac{k_0 \cdot k_c \cdot k_r}{\eta}\right)}\right] \tag{4.12}$$

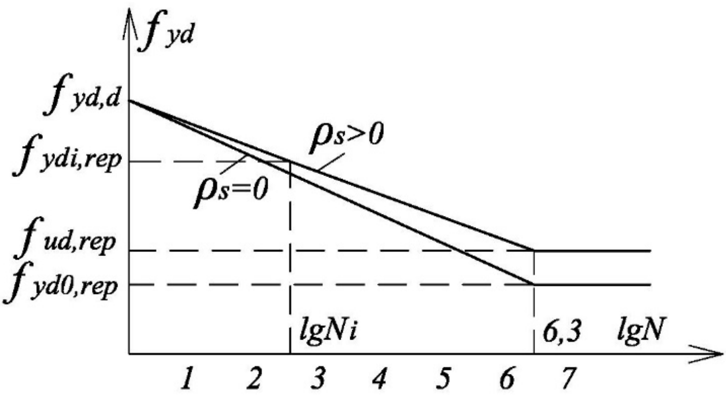

Fig. 4.2 Calculated durability reinforcement lines

In practical calculations the stress change, which occurs as a result of development of vibrocreep deformation of compressed concrete, in conditions of complex stress state, takes into account the functions of stress accumulations in concrete H_{σ_c}, longitudinal H_{σ_s} and transverse H_{σ_w} reinforcement.

These are functions of concrete vibrocreep deformations which are calculated on the theory of vibrocreep [36].

Actual stresses in concrete, longitudinal and transverse reinforcement, at the moment of time *t*, in calculations, are presented in the form:

$$\sigma_c^{max}(t)=\sigma_c^{max}(t_0)\cdot H_{\sigma_c};\quad \sigma_s^{max}(t)=\sigma_s^{max}(t_0)\cdot H_{\sigma_s};$$
$$\sigma_{sw}^{max}(t)=\sigma_{sw}^{max}(t_0)\cdot H_{\sigma_{sw}} \tag{4.13}$$

where $\sigma_c^{max}(t_0)$; $\sigma_s^{max}(t_0)$; $\sigma_{sw}^{max}(t_0)$ – initial stresses, corresponding, in concrete, longitudinal and transverse reinforcement.

In concrete and reinforced concrete elements with zero shear span a_0 / h_0 = 0, and in engineering method of durability calculation, the method of stress calculation is accepted without changes because $\sigma_{1c}^{add}(t)=0$, $\sigma_{1c}^{max}(t)=\sigma_{1c}^{max}(t_0)$ and $H_{\sigma_c}=0$, and $\sigma_{1c}^{max}(t_0)$ is defined at the first loading from equilibrium conditions on the base of fatigue destruction model. Therefore, only the right part of the condition (3.6) from p. 3, i.e. fatigue strength of concrete elements with simplifying preconditions [27] taken into account, will take the form:

$$R_{b,rep}(t)=f_{cd,rep}(t)=\frac{f_{cdti,rep}\cdot\left(\dfrac{ctg\varphi}{\delta}-\cos^2\varphi\right)}{A-G_c\cdot L_\varepsilon\cdot B\cdot\left(1/E_c+H_\varepsilon\right)} \tag{4.14}$$

and for reinforced concrete structures:

$$f_{cdi,rep}(t)=\frac{f_{ydi,rep}\cdot m\cdot A_s\cdot ctg\varphi}{H_{\sigma_s}b\cdot l_{loc}\left\langle A-\left\{G_cL_\varepsilon B+\dfrac{6E_sI_sL_\varepsilon\cdot n}{b\cdot\left(d_s\cdot\sqrt[4]{\dfrac{E_s}{E_c}}\cdot\left(1{,}4+1{,}25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right)^3}+C\right\}\cdot\left(\dfrac{1}{E_c}+H_\varepsilon\right)\right\rangle} \tag{4.15}$$

where *A*, *B*, *C* – see explanations for (3.4) and (3.5) in p. 3;

$f_{cdi,rep}$; $f_{ydi,rep}$ – fatigue strengths of concrete and reinforcement at tension;

H_ε – function of deformations accumulations in concrete at repeated loads;

E_s, E_c, G_c – modulus of elasticity of reinforcement and concrete, and concrete shear modulus.

At small shear spans, taking into account (4.1) condition (3.7) will be represented as:

$$\sigma_{1c}^{\max}(t_0) \le f_{cdc,rep}(t)/H_{\sigma_c},\ \sigma_{s\text{в}}^{\max}(t_0) \le f_{ydq,rep}(t)/H_{\sigma_s}$$
$$\sigma_s^{\max}(t_0) \le f_{yd,an}(t)/H_{\sigma_s} \tag{4.16}$$

Initial stresses $\sigma_{1c}^{\max}(t_0)$ and $\sigma_s^{\max}(t_0)$ at first load are determined from equilibrium conditions on the base of fatigue resistance model.

Fatigue strength of inclined strip in elements with transverse reinforcement and without it, after accepted simplifying preconditions, are determined by formulas:

$$f_{cdc,rep}(t) = \frac{f_{ydi,rep} \cdot m \cdot A_{sw} \cdot ctg\varphi \cdot \cos\alpha}{H_{\sigma_w} b l_{\sup} \sin\alpha \left\langle A_{\text{нп}} - \left\{ G_c L_\varepsilon B_{\text{нп}} + \dfrac{6E_s I_s \cdot L_\varepsilon \cdot n \cdot \cos(\varphi-\alpha)\sin\alpha}{\text{b}\left(d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1{,}4 + 1{,}25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right)^3 \sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon\right) \right\rangle}, \tag{4.17}$$

$$f_{cdc,rep}(t) = \frac{f_{cdti,rep} f_{cdt} \cdot \left(\dfrac{h \cdot ctg\varphi}{l_{\sup}\sin^2\alpha} - ctg\varphi \cdot ctg\alpha - \cos^2\varphi\right)}{A_{\text{нп}} - \left\{ G_c L_\varepsilon B_{\text{нп}} + \dfrac{6E_s I_s \cdot L_\varepsilon \cdot n \cdot \cos(\varphi-\alpha)\sin\alpha}{\text{b}\left(d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1{,}4 + 1{,}25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right)^3 \sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon\right)} \tag{4.18}$$

Longitudinal reinforcement fatigue strength in the place of inclined crack intersection in conditions of flat stress state, is determined as:

$$f_{ydq,rep}(t) = \frac{\sigma_u \left\langle k_{ds} - \left\{k_{ds} - k_o k_c k_r / \left[1 - \rho_s\left(k_o k_c k_r / k_{ds}\right)\right]\right\} \cdot \lg N_i / 6.3 \right\rangle}{\sqrt{1 + 3\left(\tau_s^{\max} / \sigma_{s\text{в}}^{\max}\right)^2}} \tag{4.19}$$

Longitudinal reinforcement anchoring fatigue strength has the form:

$$f_{yd,an}(t,\tau) = f_{cdi,rep} B_o (d + 2c_r) L / f_{cd} \cdot d^2 \tag{4.20}$$

where B_o – reference clutch parameter.

Fatigue strengths of concrete and reinforcement in free conditions at axle load at compression on equation of A.P. Kyrykov [35]:

$$f_{cdi,rep} = f_{cd} \left\langle k_{dc} - \left(k_{dc} - k^{a}_{c,rep} \middle/ \left\{ 1 - \rho_c \left[1 - \left(k^{a}_{c,rep} / k_{dc} \right) \right] \right\} \right) \cdot \lg N_i \middle/ 7 \right\rangle, \tag{4.21}$$

for concrete at tension:

$$f_{cdti,rep} = f_{cdt} \left\langle k_{dc} - \left(k_{dc} - k^{a}_{c,rep} \middle/ \left\{ 1 - \rho_c \left[1 - \left(k^{a}_{c,rep} / k_{dc} \right) \right] \right\} \right) \cdot \lg N_i \middle/ 7 \right\rangle, \tag{4.22}$$

for reinforcement:

$$f_{ydi,rep} = \sigma_u \left\langle k_{ds} - \left\{ k_{ds} - k_o k_c k_r / \left[1 - \rho_s \left(k_o k_{\text{CT}} k_r / k_{ds} \right) \right] \right\} \cdot \lg N_i \middle/ 6{,}3 \right\rangle \tag{4.23}$$

where k_{dc} and k_{ds} – factors of dynamic strengthening of concrete and reinforcement;

ρ_c and ρ_s – factors of stress cycle asymmetry in concrete and reinforcement;

$k^{a}_{c,rep} = f^{a}_{c,rep} / f_{cd}$ – concrete absolute fatigue strength;

$k_o = f_{yd0,rep} / \sigma_u$ – reinforcement relative fatigue strength at $\rho_s = 0$;

k_{CT} – factor that takes the presence of welding or another stress concentrator into account;

k_r – factor that takes the reinforcement diameter into account;

σ_u – reinforcement temporary resistance at its rupture.

At large shear spans taking (4.1) fatigue condition into account, (3.12) becomes:

$$\sigma^{\max}_{1c}(t_0) \le f_{cdc,rep}(t) / H_{\sigma_c}, \; \sigma^{\max}_{sw,\alpha}(t_0) \le f_{ydsw,rep}(t) / H_{\sigma_{sw}}$$
$$\sigma^{\max}_{s\mathit{в}}(t_0) \le f_{ydq,rep}(t) / H_{\sigma_s}, \; \sigma^{\max}_{s}(t_0) \le f_{ydan,rep}(t) / H_{\sigma_s} \tag{4.24}$$

Fatigue strength of compressed zone over critical inclined crack in reinforced concrete elements with transverse reinforcement and without it, after simplifying preconditions application, is determined respectively:

$$f_{cdc,rep} = \frac{f_{ydi,rep} \cdot A_{sw} \cdot ctg\varphi \cdot \cos\gamma \cdot \left(\frac{0,75h}{\lambda \cdot x_1} - 0,5\sin\varphi\cos\varphi \right)}{H_{\sigma_{w\gamma}} \cdot b \cdot s \cdot \left\langle 1 - L_{\varepsilon} \cdot \left[\frac{G_c}{\sin^2\varphi} + \frac{6E_s I_s \cdot n \cdot \cos(\varphi - \gamma)\sin\ \gamma}{b\left(d_s \cdot \sqrt[4]{\frac{E_s}{E_c}} \cdot \left(1,4 + 1,25\sqrt[4]{\frac{a_s}{d_s}} \right) \right)^3 \sin\varphi} \right] \cdot \left(\frac{1}{E_c} + H_{\varepsilon} \right) \right\rangle} \quad (4.25)$$

$$f_{cdc,rep} = \frac{f_{cdti,rep} \cdot \left(\frac{0,75h}{\lambda \cdot x_1} - 0,5\sin\varphi\cos\varphi \right) ctg\varphi}{1 - L_{\varepsilon} \cdot \left[\frac{G_c}{\sin^2\varphi} + \frac{6E_s I_s \cdot n \cdot \cos(\varphi - \gamma)\sin\ \gamma}{b\left(d_s \cdot \sqrt[4]{\frac{E_s}{E_c}} \cdot \left(1,4 + 1,25\sqrt[4]{\frac{a_s}{d_s}} \right) \right)^3 \sin\varphi} \right] \cdot \left(\frac{1}{E_c} + H_{\varepsilon} \right)} \quad (4.26)$$

Fatigue strength of longitudinal reinforcement $f_{ydsq,rep}(t)$ in the place of intersection with inclined crack, in conditions of flat stress state, is determined by (4.7). Fatigue strength of longitudinal reinforcement anchoring $f_{ydan,rep}(t)$, is determined by critical inclined crack – by (4.8), and fatigue strength if transverse reinforcement rods $f_{ydsw,rep}(t)$ at axial load is defined by (4.7), taking into account $\tau_s^{\max} = 0$.

At average shear spans taking (4.1) fatigue conditions (3.16) into account, transforms as follows:

$$\sigma_{1c}^{\max}(t_0) \le f_{cdc,rep}(t)/H_{\sigma_c},\ \sigma_{sw,\alpha}^{\max}(t_0) \le f_{ydsw,rep}(t)/H_{\sigma_{sw}},$$
$$\sigma_{s\text{в}}^{\max}(t_0) \le f_{ydsq,rep}(t)/H_{\sigma_s},\ \sigma_s^{\max}(t_0) \le f_{ydan,rep}(t)/H_{\sigma_s} \quad (4.27)$$

Fatigue strength of compressed zone over critical inclined crack in reinforced concrete elements with transverse reinforcement, and without it, after taking simplifying preconditions into account, is determined respectively:

(4.29)

$$f_{cd,rep}=\frac{f_{ydi,rep}\cdot A_{sw}\cdot\cos\beta\cdot\left(\dfrac{h\cdot ctg\varphi}{l_{\sup}\sin^2\beta}-ctg\varphi\cdot ctg\beta-\cos^2\varphi\right)}{H_{\sigma_{sw\beta}}\cdot b\cdot s\cdot\left\langle 1-L_{\varepsilon}\cdot\left(\dfrac{G_c}{\sin^2\varphi}+\dfrac{6E_sI_s\cdot n\cdot\cos(\varphi-\beta)\sin\beta}{\mathrm{b}\left(d_s\cdot\sqrt[4]{\dfrac{E_s}{E_c}}\cdot\left(1,4+1,25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right)^3\sin\varphi}\right)\cdot\left(\dfrac{1}{E_c}+H_{\varepsilon}\right)\right.}\quad(4.28)$$

$$f_{cd,rep}(t,\tau)=\frac{k_{ci,rep}f_{cdt}\cdot\left(\dfrac{h\cdot ctg\varphi}{l_{\sup}\sin^2\beta}-ctg\varphi\cdot ctg\beta-\cos^2\varphi\right)}{1-L_{\varepsilon}\cdot\left(\dfrac{G_c}{\sin^2\varphi}+\dfrac{6E_sI_s\cdot n\cdot\cos(\varphi-\beta)\sin\beta}{\mathrm{b}\left(d_s\cdot\sqrt[4]{\dfrac{E_s}{E_c}}\cdot\left(1,4+1,25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right)^3\sin\varphi}\right)\cdot\left(\dfrac{1}{E_c}+H_{\varepsilon}\right)}\quad(4.29)$$

At reinforced concrete structures design sometimes, the need for approximate estimation of reinforced concrete structures durability at transverse forces and bending moments' action, without performing any difficult calculations, appears. Applying proposed approach for determining the durability of near supports parts of reinforced concrete structures by corresponding stresses and methods of structural mechanics, we can prepare calculation dependencies for determining the corresponding boundary forces which can be carried out by given structure, at repeated load, and at the cycles number that is lower than base.

So, taking fatigue condition $\sigma_{1c}^{\max}(t)\le f_{cd,rep}(t)$ into account, boundary force that can be carried out by structure element with ***zero shear*** span at local cycle load with cycles number that is lower than the base, is determined:

- for element without reinforcement:

$$P_{lim,0}\le\frac{k_{ci,rep}\cdot f_{cdt}\left(\dfrac{ctg\varphi}{\delta}-cos^2\varphi\right)\cdot b\cdot l_{loc}}{A-G_c\cdot L_{\varepsilon}\cdot B\left(\dfrac{1}{E_c}+H_{\varepsilon}\right)}\quad(4.30)$$

- for reinforced concrete element:

$$P_{lim,0} \le \frac{k_{si,rep} \cdot \sigma_u \cdot m \cdot A_s \cdot ctg\varphi}{H_\sigma \cdot b \cdot l_{loc} \left\langle A - \left\{ G_c L_\varepsilon B + \dfrac{6E_s I_s L_\varepsilon \cdot n}{b\left[d_s \sqrt[4]{\dfrac{E}{E_c}}\left(1{,}4 + 1{,}25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right]^3} + C \right\} \left(\dfrac{1}{E_c} + H_\varepsilon\right) \right\rangle}. \tag{4.31}$$

As is known, in reinforced concrete elements with ***small shear*** span **($c_0 \le 1{,}2h_0$),** fatigue destruction in the zone of transverse forces action can take place by inclined compressed strip between support and load area, or by its stretched zone, as a result of longitudinal reinforcement rupture in the place of intersection with inclined crack, or reinforcement anchoring violation by inclined crack. To provide their bearing capacity it is necessary to adhere to corresponding fatigue conditions:

$$\sigma_{1c}^{\max}(t_0) \le f_{cd,rep}(t,\tau); \quad \sigma_{s,в}^{\max}(t_0) \le f_{ydq,rep}(t,\tau) / H_{\sigma_s};$$
$$\sigma_s^{\max}(t_0) \le f_{ydan}(t,\tau). \tag{4.32}$$

Based on these conditions, boundary forces which can be carried out by structural element with ***small shear*** span, can be determined by formulas at cycle number lower than:

- in reinforced concrete beams without transverse reinforcement:

$$P_{lim,h_0}^{min} \le \begin{cases} \dfrac{k_{ci,rep} \cdot f_{cdt} \cdot \left(\dfrac{h \cdot ctg\varphi}{l_{sup} \cdot sin^2\alpha} - ctg\varphi \cdot ctg\alpha - cos^2\varphi\right) b \cdot l_{sup} \cdot sin^2\alpha}{A_{НП} - \left\{ G_c \cdot L_\varepsilon \cdot B_{НП} + \dfrac{6E_s \cdot I_s \cdot L_\varepsilon \cdot n \cdot cos(\varphi - \alpha) sin\alpha}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1{,}4 + 1{,}25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right]^3 sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon\right)} \\ або \\ \dfrac{k_{si,rep} \cdot \sigma_u}{H_{\sigma_s}\left(\dfrac{\Omega_s \cdot a_m}{3W_s} + \dfrac{ctg\alpha}{A_s}\right)\sqrt{1 + 3\kappa_{МПС}^2}}, \\ або \quad \dfrac{k_{ci,rep} \cdot B_0 (d + 2c_r) L \cdot A_s}{d^2 \cdot ctg\alpha}; \end{cases} \tag{4.33}$$

- in reinforced concrete elements with transverse reinforcement:

$$P_{lim,h_0}^{min} \le \begin{cases} \dfrac{k_{si,rep} \cdot \sigma_u \cdot m \cdot A_{sw} \cdot ctg\varphi \cdot cos\alpha \cdot sin\alpha}{H_{\sigma_W} \left\langle A_{Н\Pi} - \left\{ G_c \cdot L_\varepsilon \cdot B_{Н\Pi} + \dfrac{6E_s \cdot I_s \cdot L_\varepsilon \cdot n \cdot cos(\varphi-\alpha) sin\alpha}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1,4+1,25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right]^3 sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon\right) \right\rangle} \\ або \\ \dfrac{k_{si,rep} \cdot \sigma_u}{H_{\sigma_s}\left(\dfrac{\Omega_s \cdot a_m}{3W_s} + \dfrac{ctg\alpha}{A_s}\right)\sqrt{1+3\cdot\kappa_{МПС}^2}}, \\ або \qquad \dfrac{k_{ci,rep} \cdot B_0 (d+2c_r) L \cdot A_s}{d^2 \cdot ctg\alpha}. \end{cases}$$

(4.34)

Taking into account fatigue conditions of concrete over danger inclined crack, longitudinal and transverse reinforcement on near support part of the beam,

$$\sigma_{1c}^{\max}(t_0) \le f_{cdc,rep}(t)/H_{\sigma_{1c}},\ \sigma_{sв}^{\max}(t_0) \le f_{ydq,rep}(t)/H_{\sigma_s},$$
$$\sigma_s^{\max}(t_0) \le f_{yd,rep}(t)/H_{\sigma_s},\ \sigma_s^{\max}(t_0) \le f_{ydan,rep}(t)/H_{\sigma_s}$$
$$\sigma_{sw,\alpha}^{\max}(t_0) \le f_{ydsw,rep}(t)/H_{\sigma_w}, \qquad (4.35)$$

we will find boundary forces that can be carried out by structural element ***with large shear span***:

- reinforced concrete beams without transverse reinforcement:

$$P_{lim,3h_0}^{min} \le \begin{cases} \dfrac{k_{ci,rep} \cdot f_{cdt} \cdot b(1+\lambda) \cdot X_1 \cdot \left(\dfrac{0,75h}{\lambda \cdot x_1} - 0,5 sin\varphi \cdot cos\varphi \right) \cdot ctg\varphi}{2H_{\sigma_{1c}} \cdot ctg\gamma \left\langle 1 - L_\varepsilon \left\{ \dfrac{G_c}{\sin^2\varphi} + \dfrac{6E_s \cdot I_s \cdot n \cdot cos(\varphi-\gamma) sin\gamma}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1,4 + 1,25\sqrt[4]{\dfrac{a_s}{d_s}} \right) \right]^3 sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon \right) \right\rangle} \\ або \\ \dfrac{k_{si,rep} \cdot \sigma_u}{H_{\sigma_s} \left(\dfrac{j_1}{A_s} + j_2 \dfrac{a_m}{3W_s^2} \right) \sqrt{1 + 3\kappa_{\sigma ПС}^2}}, \\ або \qquad \dfrac{k_{ci,rep} \cdot B_0 (d + 2c_r) L \cdot A_s}{H_{\sigma_s} \cdot d^2 \cdot j_1}, \end{cases}$$

(4.36)

Where

$$j_1 = \frac{\kappa_1 + \dfrac{(c_0 - a_1 - 0,5Xctg\alpha + \lambda \cdot z_1 sin2\varphi)}{a_1}(\kappa_1 + \kappa_2) + (1+\kappa_3) ctg\alpha}{(1+\lambda)(\kappa_1+\kappa_2)\dfrac{z_1}{a_1} + (1+\kappa_3) \cdot ctg\alpha};$$

(4.37)

$$j_2 = \frac{\kappa_2 + (1+\kappa_3)\dfrac{(c_0 - 0,5Xctg\alpha)}{z_1} \cdot ctg\alpha}{\kappa_1 + \kappa_2 + (1+\kappa_3)\dfrac{a_1}{z_1} \cdot ctg\alpha};$$

(4.38)

- reinforced concrete span elements with transverse reinforcement:

$$P_{lim,3h_0}^{min} \le \begin{cases} \dfrac{k_{si,rep} \cdot \sigma_u \cdot A_{sw}\left(1+\lambda\right) X_1 \cdot ctg\varphi \cdot sin\gamma\left(\dfrac{0,75h}{\lambda \cdot x_1} - 0,5sin\varphi \cdot cos\varphi\right)}{2 \cdot s \cdot H_{\sigma_{Wy}} \cdot H_{\sigma_{1c}} \left\langle 1 - L_\varepsilon \left\{ \dfrac{G_c}{sin^2\varphi} + \dfrac{6E_s \cdot I_s \cdot n \cdot cos(\varphi-\gamma) sin\gamma}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1,4 + 1,25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right]^3 sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon\right) \right\rangle} \\ або \\ \dfrac{1,25 \cdot k_{wi,rep} \cdot \sigma_u \cdot \omega_{sw}^1 \cdot b \cdot x}{H_{w\alpha} \cdot E_{sw} \left\{ \dfrac{\lambda \xi_1}{G_c} + \dfrac{2\lambda\left(c_0 - 0,8h_0 - 0,5l_{\sup}\right)}{E_c \cdot h_0\left[\left(1+\lambda\right) - 0,33\xi_1\left(1+\lambda+\lambda^2\right)\right]} \cdot \dfrac{h_0 - x}{x} tg\varphi \right\}}, \\ або \quad \dfrac{k_{si,rep} \cdot \sigma_u \cdot A_s}{H_{\sigma_s} \cdot j_1}, \\ або \\ \dfrac{k_{ci,rep} \cdot B_0\left(d + 2c_r\right) L \cdot A_s}{H_{\sigma_s} \cdot d^2 \cdot j_1}. \end{cases}$$

(4.39)

Based on conditions of concrete durability over danger incline crack, longitudinal and transverse reinforcement (4.36) we will find boundary forces that can be carried out by structural element with ***medium shear span***:

- reinforced concrete beams without transverse reinforcement:

$$P_{lim,2h_0}^{min} \le \begin{cases} \dfrac{k_{ci,rep} \cdot f_{cdt} \cdot \left(\dfrac{h \cdot ctg\varphi}{l_{sup} \cdot sin^2\beta} - ctg\varphi \cdot ctg\beta - cos^2\varphi\right) b \cdot l_{sup} \cdot sin^2\beta}{1 - L_\varepsilon \cdot \left\{ \dfrac{G_c}{\sin^2\varphi} + \dfrac{6E_s \cdot I_s \cdot n \cdot cos(\varphi-\beta) sin\beta}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1,4 + 1,25\sqrt[4]{\dfrac{a_s}{d_s}}\right)\right]^3 sin\varphi} \right\} \left(\dfrac{1}{E_c} + H_\varepsilon\right)} \\ або \\ \dfrac{k_{si,rep} \cdot \sigma_u}{H_{\sigma_s}\left(\dfrac{j_1}{A_s} + j_2\dfrac{a_m}{3W_s^2}\right)\sqrt{1 + 3\kappa_{\sigma ПС}^2}}, \\ або \quad \dfrac{k_{ci,rep} \cdot B_0\left(d + 2c_r\right) L \cdot A_s}{H_{\sigma_s} \cdot d^2 \cdot j_1}, \end{cases} \qquad (4.40)$$

where j_1 and j_2 determine (4.38) and (4.39) respectively;

- reinforced concrete span elements with transverse reinforcement:

$$P_{lim,2h_0}^{min} \le \begin{cases} \dfrac{k_{w\beta,rep} \cdot \sigma_u \cdot A_{sw} \cdot cos\beta \cdot \left(\dfrac{h \cdot ctg\varphi}{l_{sup} \cdot sin^2\beta} - ctg\varphi \cdot ctg\beta - cos^2\varphi \right) l_{sup} \cdot sin^2\beta}{H_{w\beta} \cdot s \left\langle 1 - L_\varepsilon \left\{ \dfrac{G_c}{sin^2\varphi} + \dfrac{6E_s \cdot I_s \cdot n \cdot cos(\varphi-\beta) sin\beta}{b\left[d_s \cdot \sqrt[4]{\dfrac{E_s}{E_c}} \cdot \left(1,1 + 1,25\sqrt[4]{\dfrac{a_s}{d_s}} \right) \right]^3 sin\varphi} \right\} \cdot \left(\dfrac{1}{E_c} + H_\varepsilon \right) \right\rangle} \\ або \\ \dfrac{k_{wi,rep} \cdot \sigma_u \cdot \omega_{sw}^1 \cdot b \cdot h_0}{H_{w\alpha} \cdot E_{sw} \cdot ctg\alpha \left\{ \dfrac{1}{G_c} + \dfrac{2\left(c_0 - x_1 ctg\alpha - 0,5 l_{sup}\right)}{E_c \cdot h_0 \left[(1+\lambda) - 0,33\xi_1 \left(1 + \lambda + \lambda^2\right) \right]} \cdot \dfrac{h_0 - x}{x} tg\varphi \right\}}, \\ або \qquad \dfrac{k_{si,rep} \cdot \sigma_u \cdot A_s}{H_{\sigma_s} \cdot j_1}, \\ або \\ \dfrac{k_{ci,rep} \cdot B_0 (d + 2c_r) L \cdot A_s}{H_{\sigma_s} \cdot d^2 \cdot j_1}. \end{cases}$$

(4.41)

For determination of boundary forces that can be carried out by structural element, unlimited number of times with reusable variable load without breaking down, in expressions (4.31), (4.32), (4.34), (4.35), (4.37), (4.40), (4.41), (4.42) should be used $k_{ci,rep} = 0,5$; and $k_{si,rep}$ – taking into account (4.10).

Relative fatigue strength of concrete at limited number of cycles $(N < 10^7)$ $k_{ci,rep} = 0,5$ is determined by (4.7), and relative fatigue strength of reinforcement $k_{si,rep}$ – by (4.12).

At low-cycle repeated load on testing beams, constituents of formulas (4.31)…(4.42) were defined by expressions:

- factor of dynamic strengthening of concrete k_d by (4.2) with the loading speed $\upsilon = 6\dfrac{кгс}{см \cdot сек} = 0,6 МПа / сек$;

- relative concrete fatigue strength $k_{ci,rep}$ at simple types of deformations (compression, tension, shear and torsion) by (4.7) taking into account the factor of stress cycle asymmetry in concrete $\rho_c = 0 \ (\rho_c = -1$ at alternating load);

- relative fatigue strength of reinforcement $k_{si,rep}$ by (4.12) taking into account values of factors $\rho_s = 0 \ (\rho_s = -1), \ \eta = 1{,}8; \ k_0 = k_c = k_r = 1$;
- functions of stress accumulations are determined in concrete H_{σ_c}, reinforcement H_{σ_s}, transverse reinforcement H_{σ_w} by expressions:

$$H_{\sigma_c} = 1 + \sigma_1^{add}(t) / \sigma_{1c}^{max}(t_0) \tag{4.42}$$

$$H_{\sigma_s} = 1 + \frac{E_s \cdot A_s \cdot L_\varepsilon \cdot H_\varepsilon}{b \cdot l_s \cdot \omega_s \cdot sin^2\varphi} \tag{4.43}$$

$$H_{\sigma_w} = 1 + \frac{0{,}5 \cdot E_s \cdot m \cdot A_{sw} \cdot L_\varepsilon \cdot H_\varepsilon}{b \cdot l_{sw} \cdot \varpi_{sw} \cdot cosa \times sin^2\varphi} \tag{4.44}$$

where $\omega_s, \ \omega_{sw}$ – stress distribution completeness coefficients respectively in longitudinal and transverse reinforcement. In the first approximation $\omega_s = \omega_{sw} = 0{,}8$. Other constituents of formulas (4.43)…(4.45) are given below;

- angle of inclination of compaction wedge faces φ at loading areas by recommendations of T.I. Baranova [32] is determined:
  before rupture crack appearance:

$$\varphi = arctg\left[0{,}48(l_{loc} / h)^{2/3}\right] \tag{4.45}$$

  after appearance and development of this crack:

$$\varphi = arctg(0{,}25 f_{cd} / f_{cdt} - 1{,}56) \tag{4.46}$$

- factor L_ε, that characterized the direction of force flow, is determined by expression:

$$\theta_н \leftrightarrow \theta_к \ L_\varepsilon = \frac{1}{\pi}\left[(2\theta_н - \pi)tg\theta_н - (2\theta_к - \pi)tg\theta_к\right] \tag{4.47}$$

where

$$\theta_н = arctgsin\varphi \cdot cos\varphi \tag{4.48}$$

$$\theta_к = arctg\left(\frac{h}{l_{sup} \cdot sin^2\alpha} - ctg\alpha\right) \text{ at } a / h_0 = 1…3, \tag{4.49}$$

$$\theta_{\kappa} = arctg\frac{h}{l_{loc}}$$ at $a / h_0 = 0$ (local compression); (4.50)

- α – angle of inclination of straight line segment that models initial part of danger inclined crack, is determined by the formula:

$$\alpha = arctg\frac{h}{c_0 - 0{,}5\left(l_{sup}^{top} + l_{sup}^{bot}\right)} \tag{4.51}$$

- $\beta(=\gamma)$ – angle of inclination of straight line segment that models end part of danger inclined crack:

$$\beta = arctg\frac{h_0}{c_0};\ \ \gamma = arctg V_{c1}^{max} / N_{c1}^{max} \tag{4.52}$$

- A, B, C – factors connected with sizes of element and load area. On $h \le 1{,}5L$ and $\delta = \dfrac{l_{loc}}{h} < 0{,}2$ $A = 1$, $B = 1 / sin^2\varphi$; $C = 0$ at $c_0 / h_0 = 0$ (local compression);
$A_{НП} = 1$, $B_{НП} = 1 / sin^2\varphi$ at $a / h_0 = 1 \ldots 3$, $l_{sup} / h < 0{,}2$; $A_{НП} = cos^2\varphi$, $B_{НП} = ctg^2\varphi$ at $l_{sup} / h \ge 0{,}2$;

- function of deformation accumulation in concrete in general form is determined by the formula:

$$H_{\varepsilon} = c_{\infty}(t,\tau)\cdot\left[1 - e^{-\gamma(t-t_0)}\right]\cdot k_{c\rho}\cdot S_k\left(\frac{\sigma_c}{f_{cd}}\right) \tag{4.53}$$

where function of concrete deformations nonlinearity:

$$S_k\left(\frac{\sigma_c^{\max}}{f_{cd}}\right) = 1 + \eta_n\left(\frac{\sigma_c^{\max}}{f_{cd}}\right)^{m_n} \tag{4.54}$$

, in which

$m_n = 5 - 0{,}07 f_{cd}$, $\eta_n = 45 / f_{cd}$ – parameters of concrete nonlinearity;

- $\varepsilon_{pl}(N)$ – nonlinear constituent of concrete vibrocreep deformations is determined by:

$$\varepsilon_{pl}(N) = \sigma_{1c}^{max}(t)\cdot H_{\varepsilon},\ \ H_{\varepsilon} = \varepsilon_{pl}(N) / \sigma_{1c}^{max}(t) \tag{4.55}$$

- $c_\infty(t,\tau)$ – boundary measure of concrete creep that is cycle loaded at the moment of τ;
- γ – researched factor that characterized the velocity of creep and vibrocreep deformations passing;
- $k_{c\rho}$ – concrete vibrocreep factor that is determined by:

$$k_{c\rho} = \rho_c + (1-\rho_c)k_{c0} \tag{4.56}$$

where k_{c0} – concrete vibrocreep factor at ρ_c =0;

- ρ – cycle asymmetry factor:

$$\rho = \frac{P_{min}}{P_{max}} \text{ – external load cycle asymmetry factor} \tag{4.57}$$

$$\rho_s = \frac{\sigma_s^{min}}{\sigma_s^{max}} \text{ – longitudinal reinforcement cycle asymmetry factor} \tag{4.58}$$

$$\rho_{sw} = \frac{\left(\frac{P_{min}}{P_{max}} + H_{\sigma w} - 1\right)}{H_{\sigma w}} \text{ – transverse reinforcement cycle asymmetry factor} \tag{4.59}$$

$$\rho_c = \frac{\sigma_c^{min}}{\sigma_c^{max}} \text{ – concrete cycle asymmetry factor} \tag{4.60}$$

- n – number of transverse rods in shear span;
- a_s – protective layer of working reinforcement of concrete;
- d_s – diameter of the working reinforcement;

For given design of testing beams, minimum values of calculated boundary forces (Table 4.1) take place at condition of exhaustion of concrete durability over danger inclined crack and transverse reinforcement in shear span according to

Table 4.1 Calculation values of boundary forces of near support areas durability of testing samples-beams, destructive transverse force and their ratio according to the experiment plan

№ of test	Natural and coded values of testing factors								Boundary forces at durability condition			Destructive transverse force (V_{u3}, kN) after sample uploading	$k_{cycl} = V_{u3} / P_{lim,c,sw}$	$\hat{k}_{cycl} = V_{u3} / P_{im,c,sw}$ for (4.63)
	Coded values				Natural values				Concrete over inclined crack and transverse reinforcement, $P_{lim,c,sw}$, kN	Working reinforcement on inclined crack, $Pl_{im,s}$, kN	Clutch of working reinforcement with concrete behind support, $P_{lim,an}$, kN			
	X_1	X_2	X_3	X_4	a/h_0	C, MPa	ρ_{sw} (Bpl)	η						
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	+	+	+	+	3	C40/50	0,0044	0…0,8	29,77	105,6	120,4	66	2,22	2,10
2	+	+	+	−	3	C40/50	0,0044	0…0,5	29,77	105,6	120,4	68	2,28	2,34
3	+	+	−	+	3	C40/50	0,0016	0…0,8	10,72	105,6	120,4	52	4,85	4,96
4	+	+	−	−	3	C40/50	0,0016	0…0,5	10,72	105,6	120,4	54	5,04	5,20
5	+	−	+	+	3	C16/20	0,0044	0…0,8	17,52	113,7	129,7	62	3,54	3,66
6	+	−	+	−	3	C16/20	0,0044	0…0,5	17,52	113,7	129,7	64	3,65	3,90
7	+	−	−	+	3	C16/20	0,0016	0…0,8	6,31	113,7	129,7	48	7,61	7,52
8	+	-	-	-	3	C16/20	0,0016	0…0,5	6,31	113,7	129,7	50	7,92	7,76
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
9	−	+	+	+	1	C40/50	0,0044	0…0,8	78,54	172,2	395,9	150	1,91	1,90
10	−	+	+	−	1	C40/50	0,0044	0…0,5	78,54	172,2	395,9	160	2,04	2,14
11	−	+	−	+	1	C40/50	0,0016	0…0,8	28,26	172,2	395,9	136	4,81	4,76

12	−	+	−	−	1	C40/50	0,0016	0…0,5	28,26	172,2	395,9	146	5,17	5,00
13	−	−	+	+	1	C16/20	0,0044	0…0,8	48,93	433,5	395,9	114	2,33	2,10
14	−	−	+	−	1	C16/20	0,0044	0…0,5	48,93	433,5	395,9	124	2,53	2,34
15	−	−	−	+	1	C16/20	0,0016	0…0,8	17,61	433,5	395,9	100	5,68	5,96
16	−	−	−	−	1	C16/20	0,0016	0…0,5	17,61	433,5	395,9	110	6,25	6,20
17	+	0	0	0	3	C30/35	0,0029	0…0,65	15,03	109,1	124,4	72	4,79	4,39
18	−	0	0	0	1	C30/35	0,0029	0…0,65	46,32	253,7	395,9	144	3,11	3,51
19	0	+	0	0	2	C40/50	0,0029	0…0,65	28,66	105,6	121,3	94	3,28	3,21
20	0	−	0	0	2	C16/20	0,0029	0…0,65	16,37	113,7	129,9	74	4,52	4,59
21	0	0	+	0	2	C30/35	0,0044	0…0,65	36,77	109,1	125,1	91	2,47	2,47
22	0	0	−	0	2	C30/35	0,0016	0…0,65	13,24	109,1	125,1	77	5,82	5,83
23	0	0	0	+	2	C30/35	0,0029	0…0,8	23,53	109,1	125,1	85	3,61	3,64
24	0	0	0	−	2	C30/35	0,0029	0…0,5	23,53	109,1	125,1	92	3,91	3,88
25	0	0	0	0	2	C30/35	0,0029	0…0,65	23,53	109,1	125,1	88	3,74	3,84
$\sum(\bar{y}-\hat{y})^2$												0,930		
$\sigma=\sqrt{\sum(\bar{y}\cdot\hat{y})^2/24}$												0,1970		
$\mho=(\sigma/b_0)\cdot 100,\%$												5,1%		

described physic models of fatigue destruction that completely converges with real character of destruction of their near support areas at repeated transverse loads, in Chapter 3. These calculations show (Table 4.1) that fatigue destruction of longitudinal working reinforcement of testing samples, as a result of their rupture or slipping along danger inclined crack, is impossible, due to their sufficient amount and appropriate anchoring on support and behind them.

Comparison of calculation and actual values of bearing capacity of inclined sections of samples-beams showed that, despite the complete coincidence of physic models and actual pictures of destructions of their near support areas, the destructive uploading transverse force V_{u3} is 1,91…7,92 times higher than the predicted transverse force of concrete durability over danger inclined crack and transverse reinforcement $P_{lim,c,sw}$, determined by formulas (4.34)…(4.42) and taking recommendations of I.T. Mirsayapov [27] and A.P. Kyrylov [35] into account. The reason of such misconduct is higher destructive influence of repeated cycle load, compared to low-cycle repeated transverse load, undervaluation of actual influence of different design factors in indicated works, and the fact that, at indicated levels of load, there was no destruction of beams.

For this problem solution on common stage of research boundary forces P^{min}_{lim,h_0}; $P^{min}_{lim,2h_0}$; $P^{min}_{lim,3h_0}$ in expressions (4.34)…(4.42), it is recommended to multiply on the factor k_{cycl}. That is the resistance of near support areas of span reinforced concrete structures, in particular, calculation values of transverse force V_{Rd}, which can be carried by inclined section of such structure with transverse reinforcement or without it, with small, middle or large shear span. It is expedient to determine it by taking into account its durability at low-cycle repeated load action by expression:

$$V_{Rd} = k_{cycl} \begin{Bmatrix} P^{min}_{lim,h_0} \\ P^{min}_{lim,2h_0} \\ P^{min}_{lim,3h_0} \end{Bmatrix} \tag{4.61}$$

where k_{cycl} − ratio of destructive transverse forces to minimum calculation boundary forces of durability of near support areas of testing samples-beams, calculated by formulas (4.34)…(4.42), using characteristic values of concrete and reinforcement strength i.e. prism concrete strength and yield strength of the reinforcement.

Mathematical model of factor $\hat{k}_{cycl}$, that characterizes the ratio of destructive (uploading) transverse force V_{u3} to calculation value of boundary force, at a condition of concrete durability over danger inclined crack and transverse reinforcement in shear span $P_{lim,c,sw}$ at low-cycle repeated load of testing samples-beams, has the form:

$$\hat{Y}(k_{cycl}) = 3{,}84 + 0{,}44X_1 - 0{,}69X_2 - 1{,}68X_3 - 0{,}12X_4 + 0{,}11X_1^2 + 0{,}06X_2^2 +$$
$$+0{,}31X_3^2 - 0{,}08X_4^2 - 0{,}34X_1X_2 + 0{,}25X_2X_3, \qquad \upsilon = 5{,}1\%. \tag{4.62}$$

The highest influence on the value of factor k_{cycl} (Fig. 4.3) has the number of transverse reinforcement (factor X_3), then – concrete class (X_2), then – value of relative shear span (X_1) and, finally, − the level of low-cycle repeated load (X_4). So, the value $\hat{k}_{cycl}$ will decrease (calculation and actual bearing capacity of near support areas of reinforced concrete beams will converge) relative to its average value 3,84:

- at increasing the number of transverse reinforcement ρ_{sw} from 0,0016 to 0,0044, concrete class from C16/20 to C40/50, level of low-cycle repeated load η from 0…0,5 to 0…0,8, respectively, on 88, 36 and 6%;
- at decreasing the relative shear span a/h_0 from 3 to 1 on 23%.

Presence of quadratic effects at testing factors, proves that following increasing the number of transverse reinforcement, concrete beyond its changes, will not lead to significant reduction of factor k_{cycl}, and following increasing the shear span and reduction of level of repeated load, will lead to their significant increase.

Relative shear span, concrete class and number of transverse reinforcement interact substantially. So, k_{cycl} will be decreasing with simultaneous reduction of shear span and increase of concrete class, increase of number of transverse reinforcement and reduction of concrete class.

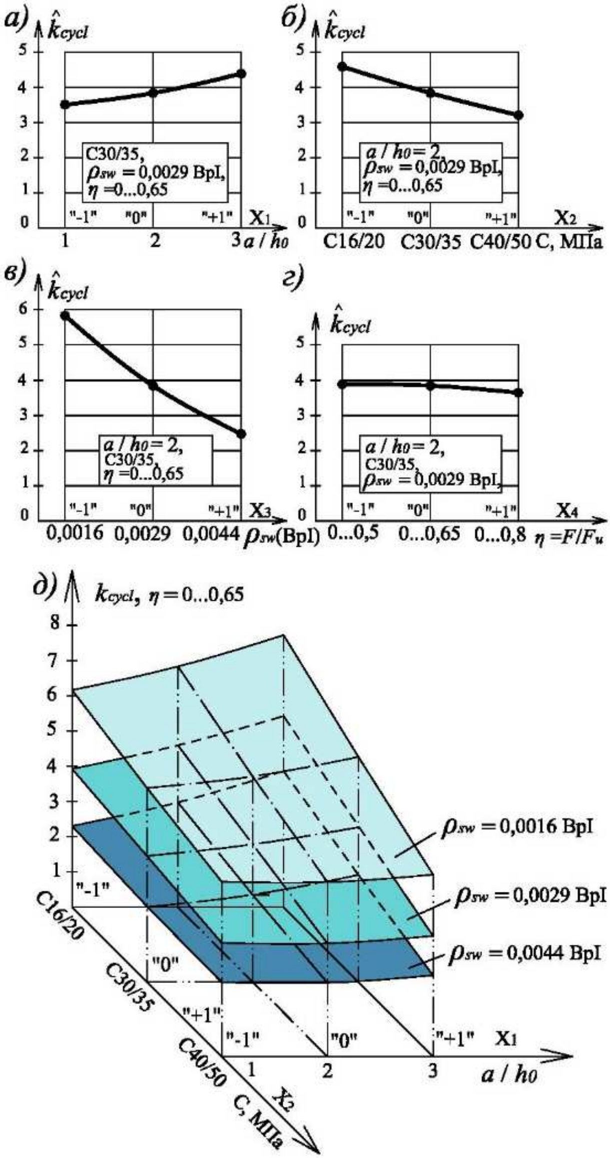

Fig 1.5. Fig. 4.3 Factor k_{cycl} dependency on relative shear span (a), concrete class (b), number of transverse reinforcement (c), level of low-repeated load of constant sign (d) and joint influence of testing factors on it

For convenience of practical application of factor k_{cycl}, it is expediently to change a form of relation (4.63) by changing coding values of testing factors with natural ones:

$$k_{cycl} = 3,84 + 0,44\left(\frac{a/h_0 - 2}{1}\right) - 0,69\left(\frac{C-35}{15}\right) - 1,68\left(\frac{\rho_{sw} - 0,0029}{0,0014}\right) -$$
$$-0,12\left(\frac{\eta - 0,65}{0,15}\right) + 0,11\left(\frac{a/h_0 - 2}{1}\right)^2 + 0,06\left(\frac{C-35}{15}\right)^2 + 0,31\left(\frac{\rho_{sw} - 0,0029}{0,0014}\right)^2 -$$
$$-0,08\left(\frac{\eta - 0,65}{0,15}\right)^2 - 0,34\left(\frac{a/h_0 - 2}{1}\right)\left(\frac{C-35}{15}\right) + 0,25\left(\frac{C-35}{15}\right)\left(\frac{\rho_{sw} - 0,0029}{0,0014}\right) \quad (4.63)$$

Relation (4.63) is fair, not only inside the change of testing factors, but its extrapolation is possible on the value up to 25% from the values of their intervals.

For convergence of the test and calculation values of uploading transverse force at stabilized low-cycle, repeated load calculation boundary forces of concrete durability, transverse and longitudinal reinforcement, calculated with taking into account the repeated cycle transverse load based on theories of A.P. Kyrylov [35] and I.T. Mirsayapov [27], it is expediently to increase, as a way of introduction of calculation relation (4.63), factor k_{cycl}, which comprehensively takes into account the influence of the most important constructive factors and level of repeated low-cycle load, both, individually and in interaction with each other.

4.1 Conclusions on chapter 4:

1. Presented engineering method takes into account change of stress-strain state of span reinforced concrete structures at their repeated load, as well as the change of strength properties of concrete, reinforcement and their clutch, at the moment *t,* and can be used on all range of strength characteristics of materials change- from low-cycle repeated, to repeated cycle load;

2. Proposed engineering method of near support areas strength calculation of beam reinforced concrete structures provides all possible destruction schemes, and allows to predict fatigue strength of the concrete over danger inclined crack, and presents transverse reinforcement, longitudinal working reinforcement on rupture or slippage on this crack because of its inadequate amount (break) or its inadequate anchorage.

GENERAL CONCLUSIONS:

1) Generalized model of the element deformation should be able to equally reflect, both, the nature of the growth of relative deformation of materials, and a process of continuous redistribution of stresses in them. So, the real state of the reinforced concrete structure can only be reflected when used in conjunction with, both, stress and deformations diagrams.
2) Accepted in general case, nonlinear deformational model of bar construction allows to form the union positions of reinforced concrete mechanics, considers features of mutual work of concrete and reinforcement on all stages, including destruction, in its calculated cross sections in the general case of a stressed state, taking into account the joint action of longitudinal and transverse forces, bending and torque. It can be used for designing or reinforcing beams, crossbars, columns and trusses of a rectangular cross-section, as well as checking the bearing capacity of existing core reinforced concrete structures operating in conditions of complex stress-strain state.
3) Described in Chapter 3, physical models and calculation schemes of near support areas resistance of not over reinforced span reinforced concrete structures to repeated load of high level, different types of fatigue destruction of materials considering vibro-creep deformations, accumulation of damages in form of fatigue micro- and macro cracks, are envisaged.
4) Presented engineering method takes into account the change of stress-strain state of span reinforced concrete structures at their repeated load, as well as, a change of strength properties of the concrete, reinforcement and their clutch at the moment *t,* and can be used on all range of strength characteristics of materials change- from low-cycle repeated, to repeated cycle load.

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