ON THE DYNAMICS OF NON-RIGID ASTEROID ROTATION

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We have presented in this communication a new solving procedure for the dynamics of *non-rigid* asteroid rotation, considering the final spin state of rotation for a small celestial body (asteroid). The last condition means the ultimate absence of the applied external torques (including short-term effect from torques during collisions, long-term YORP effect, etc.).

Fundamental law of angular momentum conservation has been used for the aforementioned solving procedure. The system of *Euler* equations for dynamics of *non-rigid* asteroid rotation has been explored with regard to the existence of an analytic way of presentation of the approximated solution.

Despite of various perturbations (such as collisions, YORP effect) which destabilize the rotation of asteroid via deviating from the current spin state, the inelastic (mainly, tidal) dissipation reduces kinetic energy of asteroid. So, evolution of the spinning asteroid should be resulting by the rotation about maximal-inertia axis with the proper spin state corresponding to minimal energy with a fixed angular momentum.

Basing on the aforesaid assumption (component K_1 is supposed to be fluctuating near the given appropriate constant of the fixed angular momentum), we have obtained that 2-nd component K_2 is the solution of appropriate *Riccati* ordinary differential equation of 1-st order, whereas component K_3 should be determined via expression for K_2.

Keywords: Tidal dissipation, asteroid rotation, angular momentum.

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1. Introduction, the system of equations.

The main motivation of the current research is the analytical exploration of the dynamics of asteroid rotation when it moves in elliptic orbit through Space. In our previous research [1], we have explored the regimes of *rigid* asteroid rotation under the additional influence of YORP-effect. Let us note that assumption of asteroid rotating as rigid body means that distances between various points inside the rigid body should be preferably constant or should be elongated negligibly.

We will consider here regime of rotation of small celestial bodies (asteroids less than < 10 km in diameter) which differ from the rigid body in a sufficient extent. It means that distances between various points inside the asteroid can not be considered as being elongated negligibly. Meanwhile, only circa 20% of all the registered asteroids (near-Earth objects in NASA data base) are recognized as to be close to the rigid body approximation. For example, we can provide the comparison with to the actual/observed data for nonrigid asteroids, which is available in the modern research with respect to the *rubble pile asteroids* [2].

It is very important to create the adequate physical model along with the mathematical model of the aforementioned asteroid's spinning phenomenon with the main aim of the clarifying the results of data of astrometric observations. Indeed, if regime of rotation of asteroid is suddenly changing, we could observe even physical disintegration of asteroids (or self-destruction under the influence of sudden acceleration during a fast rotation [3]).

It is also worth to note that fundamental law of angular momentum conservation should be valid even during the *non-rigid* regime of asteroid's rotation [4]. Namely, theorem of conservation of angular momentum describes rotation of asteroid in a frame of reference fixed in the rotating body [5] ($I_i \neq 0$):

$$\frac{d\vec{K}}{dt} + [\vec{\Omega} \times \vec{K}] = \vec{M}\,, \tag{1}$$

where $\vec{K} = \{ I_i \cdot \Omega_i \}$, whereas $\vec{\Omega} = \{ \Omega_i \}$ (here Ω_i are the components of angular velocity vector along the principal axes, i = 1, 2, 3), I_i are the principal moments of inertia, and $\vec{M} = \vec{M}(t)$ is the total sum of applied external torques (including short-term effect from torques during collisions, long-term YORP effect [1], [6], etc.).

Let us especially empasize that we will consider here principal moments of inertia to be variable (time-dependent, $I_i = I_i(t)$), in general case; e.g., components of inertia tensor of asteroid may be changed during collisions [4]. Indeed, we should take into consideration the possible changes in its form, along with the decreasing of the mass via partial physical disintegration of asteroids during collisions or even via self-destruction due to the regime of fast rotation [1]. Despite of various perturbations (such as collisions, YORP effect) which destabilize the rotation of asteroid via deviating from the current spin state, the inelastic (mainly, tidal) dissipation [7-9] reduces kinetic energy of asteroid.

It means that evolution of the spinning asteroid should be resulting by the rotation about maximal-inertia axis [7] with the proper spin state corresponding to minimal energy with a fixed angular momentum.

We will consider in (1) only such the aforesaid final dynamical state of asteroid rotation (which is fluctuating near the given appropriate constant of the fixed angular momentum). Asteroid is supposed to be moving along its orbit far from the close influences of additional gravitational forces from planet of mass m_{planet} or far from *Hill sphere* [1] (motion of asteroid is determined by equations of ER3BP with primaries m_{planet} and M_{Sun}, $m_{planet} < M_{Sun}$):

$$r_H = a_p \cdot \left(\frac{m_{planet}}{M_{Sun}} \right)^{\frac{1}{3}} \qquad (*)$$

where a_p is semimajor axis of the planet.

Let us also assume (as first approximation) that all external torques, associated with inertial forces, tides, YORP effect are neglected in (1) (i.e., $\vec{M} \simeq \vec{0}$ in (1)).

According to the results of [7], inelastic (mainly tidal) dissipation, which is reducing kinetic energy, yields evolution of spin towards rotation about maximal-inertia axis I_1 with rate of rotation Ω_1 (for definiteness, $I_1 > I_2 > I_3$); it means:

$$\{\Omega_2, \Omega_3\} << \Omega_1 \;\; \Rightarrow \;\; \{\Omega_2, \Omega_3\} \to 0 \qquad (2)$$

The last but not least, let us additionally note that the spatial ER3BP is not conservative, and no integrals of motion are known [10] (including total angular momentum, which combines the expressions in (1) and orbital angular momentum).

2. Analytical exploring of the system of equations (1).

First of all, we should note that (1) is the system of 3 nonlinear differential equations with respect to $\vec{K} = \{ I_i \cdot \Omega_i \}$ (with all coefficients depending on time *t*):

$$\frac{d\vec{K}}{dt} + [\vec{\Omega} \times \vec{K}] = \vec{0}\,, \quad \Rightarrow \quad \begin{cases} \dfrac{dK_1}{dt} = K_2 \cdot (\Omega_3) - K_3 \cdot (\Omega_2)\,, \\[2ex] \dfrac{dK_2}{dt} = K_3 \cdot (\Omega_1) - K_1 \cdot (\Omega_3)\,, \\[2ex] \dfrac{dK_3}{dt} = K_1 \cdot (\Omega_2) - K_2 \cdot (\Omega_1)\ . \end{cases} \tag{3}$$

$$\Rightarrow \begin{cases} \dfrac{dK_1}{dt} = K_2 \cdot (\dfrac{K_3}{I_3}) - K_3 \cdot (\dfrac{K_2}{I_2})\,, \\[2ex] \dfrac{dK_2}{dt} = K_3 \cdot (\dfrac{K_1}{I_1}) - K_1 \cdot (\dfrac{K_3}{I_3})\,, \\[2ex] \dfrac{dK_3}{dt} = K_1 \cdot (\dfrac{K_2}{I_2}) - K_2 \cdot (\dfrac{K_1}{I_1})\ . \end{cases} \quad \Rightarrow \quad \begin{cases} \dfrac{dK_1}{dt} = K_2 \cdot K_3 \cdot \left(\dfrac{I_2 - I_3}{I_2 \cdot I_3} \right), \\[2ex] \dfrac{dK_2}{dt} = K_1 \cdot K_3 \cdot \left(\dfrac{I_3 - I_1}{I_1 \cdot I_3} \right), \\[2ex] \dfrac{dK_3}{dt} = K_1 \cdot K_2 \cdot \left(\dfrac{I_1 - I_2}{I_1 \cdot I_2} \right)\ . \end{cases} \tag{4}$$

Let us multiply 1-st equation of system (3) or (4) on (K_1 / I_1), the 2-nd Eq. on (K_2 / I_2), 3-rd on (K_3 / I_3); then if we sum all the resulting equations of the system above, we should obtain

$$\left(\frac{1}{I_1} \right) \frac{d(K_1^{\,2})}{dt} + \left(\frac{1}{I_2} \right) \frac{d(K_2^{\,2})}{dt} + \left(\frac{1}{I_3} \right) \frac{d(K_3^{\,2})}{dt} = 0 \tag{5}$$

3. Solving procedure and the approximated solution for Eqns. (1).

According to the assumption (2) above, in (1) we will consider only the final dynamical state of asteroid rotation (which is fluctuating near the given appropriate constant of the fixed angular momentum, $K_1 \cong const$). It means that equation (5) can be transformed to the form below

$$\frac{d\,(K_2^{\,2})}{d\,t} + \left(\frac{I_2}{I_3}\right)\frac{d\,(K_3^{\,2})}{d\,t} \cong 0 \qquad (6)$$

Let us note that in case $K_1 \cong const$, 1-st equation of system (4) should be satisfied accordingly at first approximation (if we take into account assumption (2) for the right part of 1-st equation of system (4), where $\Omega_2 = (K_2 / I_2)$, $\Omega_3 = (K_3 / I_3)$, $\{\Omega_2, \Omega_3\} \to 0$).

As for the 2-nd equation of system (4), we obtain (here below $K_1 \cong const$):

$$K_3 = \frac{1}{K_1}\left(\frac{I_1 \cdot I_3}{I_3 - I_1}\right)\frac{d\,K_2}{d\,t} \qquad (7)$$

Now, as for the 3-rd equation of system (4), let us substitute expression for K_3 from the 2-nd Eqn. of (4) the expression for derivative in the left part; it yields as below

$$\left(\frac{I_1 \cdot I_3}{I_3 - I_1}\right)\cdot\frac{d^2 K_2}{d\,t^2} + \frac{d\left(\dfrac{I_1 \cdot I_3}{I_3 - I_1}\right)}{d\,t}\cdot\frac{d\,K_2}{d\,t} - \left(K_1^{\,2}\cdot\left(\frac{I_1 - I_2}{I_1 \cdot I_2}\right)\right)\cdot K_2 = 0 \qquad (8)$$

where equation (8) for the dynamics of component $K_2 = I_2 \cdot \Omega_2$ could be transformed by change of variables $y = (K_2' / K_2)$ to the *Riccati* ODE of 1-st order [1].

Discussion

We have explored here the dynamics of *non-rigid* asteroid rotation, considering the final spin state of rotation for a small celestial body (asteroid). *Non-rigid* character of asteroid rotation means that principal moments of inertia are variable (time-dependent, $I_i = I_i(t)$, i = 1, 2, 3).

Fundamental law of angular momentum $\vec{K} = \{ I_i \cdot \Omega_i \}$ conservation (which should be valid even during the *non-rigid* regime of asteroid's rotation) has been used at obtaining the analytical algorithm for solving. The proper approximate solution has been obtained which is presented below:

- component $K_1 = I_1(t) \cdot \Omega_1(t)$ is supposed to be fluctuating near the given appropriate constant of the fixed angular momentum, $K_1 \cong const$;

- component $K_2 = I_2(t) \cdot \Omega_2(t)$ is the solution of the appropriate *Riccati* ODE (8):

$$\left(\frac{I_1 \cdot I_3}{I_3 - I_1}\right) \cdot \frac{d^2 K_2}{d t^2} + \frac{d\left(\dfrac{I_1 \cdot I_3}{I_3 - I_1}\right)}{d t} \cdot \frac{d K_2}{d t} - \left(K_1^2 \cdot \left(\frac{I_1 - I_2}{I_1 \cdot I_2}\right)\right) \cdot K_2 = 0,$$

- component $K_3 = I_3(t) \cdot \Omega_3(t)$ is determined in (7) via expression for K_2:

$$K_3 = \frac{1}{K_1}\left(\frac{I_1 \cdot I_3}{I_3 - I_1}\right)\frac{d K_2}{d t}$$

We should additionally note that for reason of a special character of the solutions of *Riccati*-type ODEs, there exists a possibility for sudden *jumping* of magnitude of the solution at some meaning of time-parameter t [11-15].

Mathematical procedure of presenting the components of angular velocity via Euler angles [16] (and Wisdom angles [17]) has been demonstrated at the Appendix in [1].

Conclusion

We have presented in this communication a new solving procedure for the dynamics of *non-rigid* asteroid rotation, considering the final spin state of rotation for a small celestial body (asteroid). The last condition means the ultimate absence of the applied external torques (including short-term effect from torques during collisions, long-term YORP effect, etc.).

Fundamental law of angular momentum conservation has been used for the aforementioned solving procedure. The system of *Euler* equations for dynamics of *non-rigid* asteroid rotation has been explored with regard to the existence of an analytic way of presentation of the approximated solution.

Despite of various perturbations (such as collisions, YORP effect) which destabilize the rotation of asteroid via deviating from the current spin state, the inelastic (mainly, tidal) dissipation reduces kinetic energy of asteroid. So, evolution of the spinning asteroid should be resulting by the rotation about maximal-inertia axis with the proper spin state corresponding to minimal energy with a fixed angular momentum.

Basing on the aforesaid assumption (component K_1 is supposed to be fluctuating near the given appropriate constant of the fixed angular momentum), we have obtained that 2-nd component K_2 is the solution of the appropriate *Riccati* ordinary differential equation of 1-st order, whereas component K_3 should be determined via expression for K_2.

There is additional condition for obtaining such approximated solution ($I_1 > I_2 \geq I_3$):

$$\{\Omega_2, \Omega_3\} << \Omega_1 ,$$

where Ω_i are the components of angular velocity vector along the principal axes (i = 1,2,3), I_i are the principal moments of inertia.

The last but not least, we can obtain one additional class of approximated solutions of system (1) with non-zero external applied torques $\vec{M}(t) \neq \vec{0}$; mathematical procedure of obtaining such the additional solution has been moved to an Appendix, with only the resulting formulae left in the main text (here below $K_1 \cong const$):

$$K_2 \cdot \frac{d K_2}{d t} \;=\; M_2 \cdot K_2 + \left\{ M_1 \cdot K_1 \cdot \left(\frac{I_2}{I_3 - I_2} \right) \cdot \left(\frac{I_3 - I_1}{I_1} \right) \right\} , \qquad (10)$$

$$K_3 \cdot \frac{d K_3}{d t} \;=\; M_3 \cdot K_3 + \left\{ K_1 \cdot M_1 \cdot \left(\frac{I_3}{I_3 - I_2} \right) \cdot \left(\frac{I_1 - I_2}{I_1} \right) \right\} , \qquad (11)$$

where equations (10)-(11) for the dynamics of components of angular momentum K_2, K_3 ($K_2 = I_2 \cdot \Omega_2$, $K_3 = I_3 \cdot \Omega_3$) are the *Abel* ODEs of 1-st order of the 2-nd kind [13].

Also, the remarkable articles [18-20] should be cited, which concern the problem under consideration.

Appendix (additional class of approximated solutions of system (1)).

Let us obtain the additional class of approximated solutions of system (1). We consider the final dynamical state of asteroid rotation (which is fluctuating near the given appropriate constant of the fixed angular momentum, $K_1 \cong$ *const*) for which we assume $\vec{M}(t) \neq \vec{0}$.

Then 1-st equation of system (1) should be satisfied accordingly (at first approximation) under the appropriate condition below:

$$K_2 \cdot K_3 \cdot \left(\frac{I_3 - I_2}{I_2 \cdot I_3} \right) \cong M_1 \qquad (9)$$

Meanwhile, there is no need to take into account assumption (2) for the right part of 1-st equation of system (1) in this case.

As for the 2-nd and 3-rd equations of system (1), we obtain (here below $K_1 \cong$ *const*):

$$K_2 \cdot \frac{d K_2}{d t} = M_2 \cdot K_2 + \left\{ M_1 \cdot K_1 \cdot \left(\frac{I_2}{I_3 - I_2} \right) \cdot \left(\frac{I_3 - I_1}{I_1} \right) \right\} , \tag{10}$$

$$K_3 \cdot \frac{d K_3}{d t} = M_3 \cdot K_3 + \left\{ K_1 \cdot M_1 \cdot \left(\frac{I_3}{I_3 - I_2} \right) \cdot \left(\frac{I_1 - I_2}{I_1} \right) \right\} , \tag{11}$$

where equations (10)-(11) for the dynamics of components of angular momentum K_2, K_3 ($K_2 = I_2 \cdot \Omega_2$, $K_3 = I_3 \cdot \Omega_3$) are the *Abel* ODEs of 1-st order of the 2-nd kind [13]. These Eqns. can be transformed by the appropriate change of variables $K_3 = 1/u$ to the *Abel* ODEs of the 1-st kind (of *Riccati* type).

Accordingly, for the aforesaid reason of a special character of the solutions of *Riccati*-type ODEs (see **Discussion**), there exists a possibility for sudden *jumping* of magnitude of the solution at definite meaning of time-parameter t [11-15].

In the physical sense, such jumping of *Riccati*-type solutions of Eqn. (8) can be associated with the effect of sudden acceleration/deceleration of angular velocity's component Ω_2 at definite moment of time t_0 (or with the alternative effect of crucial changes in the principal moment of inertia $I_2(t)$ of asteroid during the process of rotation).

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Conflict of interest

Authors declare that there is no conflict of interests regarding publication of article.

Remark regarding contributions of authors as below:

In this research, Dr. Sergey Ershkov is responsible for the general ansatz and the solving procedure, simple algebra manipulations, calculations, results of the article in Sections 1-3 and also is responsible for the search of approximate solutions.

Dr. Dmytro Leshchenko is responsible for theoretical investigations as well as for the deep survey in literature on the problem under consideration (in Section 1, see the remark regarding ref. [2]).

Both authors agreed with the results and conclusions of each other in Sections 1-3.

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