ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF SIGNIFICANTLY NONLINEAR DIFFERENTIAL EQUATIONS OF THE THIRD ORDER

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A third-order differential equation is considered

$$y''' = \alpha_0 p(t)\varphi(y), \quad (1)$$

where $\alpha_0 \in \{-1,1\}$, $p:[a,\omega[\to]0,+\infty[$ $(-\infty < a < \omega \le +\infty)$ is acontinuous function, $\varphi:\Delta_{Y_0} \to]0,+\infty[$ is acontinuous functionregularly varying of order σ as $y \to Y_0$ such, that $\lim\limits_{y\to Y_0,\, y\in\Delta_{Y_0}} \varphi(y) = \{0;+\infty\}$, here$Y_0$ is zero or $\pm\infty$, Δ_{Y_0} is a one-sided neighborhood of the pointY_0. In thecase of $\sigma \neq 1$, this equation is substantiallynonlinear andasymptotically closeat $y \to Y_0$ to the Emden-Fowler type equation.This case is considered provided that the function $\varphi$is twice continuously differentiated and satisfies the conditions

$$\varphi'(y) \neq 0, \quad \lim_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \varphi(y) = \begin{cases} \text{или } 0, \\ \text{или } +\infty, \end{cases} \quad \lim_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \frac{y\varphi''(y)}{\varphi'(y)} = \sigma - 1, \qquad (2)$$

from which, in particular, it follows that $\lim\limits_{y\to Y_0,\, y\in\Delta_{Y_0}} \frac{y\varphi'(y)}{\varphi(y)} = \sigma$ and istherefore φ a normalized properly variable function of order σ at $y \to Y_0$.

From the works of V.M. Evtukhov, which deal with differential equations with power nonlinearities, it follows that the set of all $P_\omega(Y_0, \lambda_0)$-solutions of the investigated third order diffrential equation breaks down by its asymptotic properties into five different types depending on the following values of the parameter $\lambda_0 : \lambda_0 \in \mathbb{R} \setminus \{0, \frac{1}{2}, 1\}$(not a specialcase), $\lambda_0 = \pm\infty$, $\lambda_0 = 0, \lambda_0 = \frac{1}{2}, \lambda_0 = 1$(special cases). For each of the indicated values λ_0, the necessary and sufficient conditions for the existence of this $P_\omega(Y_0, \lambda_0)$-solution of the equation are obtained, the question of their number is clarified, asymptotic representations of these solutions and their derivatives are written up to the second order inclusive. The developed technique made it possible to describe the asymptotics of not only correct, but also singular solutions, the question of the existence and number of singular solutions was studied.

Bibliography

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