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Perturbed rotational motions of a rigid body, close to the Lagrange case, under the action of unsteady restoring and perturbation torques

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The problem of evolution of the rigid body rotation about a fixed point continues to attract the attention of researchers. In the aspect of applications, the analysis of rotational motion of bodies about a fixed point is important for solving the problems of astronautics, the problems of entry the flying vehicles into the atmosphere and dynamics of the rotating projectile and gyroscopy. A perturbed motions of the rigid body (close to Lagrange's case) have been considered in [1, 2]. Here the appropriate conditions were presented for the possibility of applying the averaging procedure for the equations of motions with respect to the phase of nutation angle, the averaged system of equations was obtained.

We consider the rotation of a dynamically symmetric rigid body about a fixed point under the action of the restoring torque and perturbations depending on the slow time. We set the problem of investigating the asymptotic behavior of the solutions of system for a small parameter; the analysis will be carried out by method of averaging. We receive from the equations of motion for the unperturbed system at parameter $\varepsilon = 0$ the first integrals: G_z is the projection of the angular momentum vector onto vertical axis Oz; H is the total energy of the body; r is the projection of the angular velocity vector onto the axis of dynamical symmetry.

We express the nutation angle θ in case of unperturbed motion as function of the time t , the integrals of motion and arbitrary phase constant β as below

$$\cos\theta = u_1 + (u_2 - u_1)\,\mathrm{sn}\,(\alpha t + \beta)\,, k^2 = (u_2 - u_1)\,/\,(u_3 - u_1)\,,$$

$$\alpha = \sqrt{\mu\,(u_2 - u_1)\,(2A)^{-1}}, 0 \le k^2 \le 1.$$

Here, $\cos\theta$ is the periodic function $(\alpha t + \beta)$ with a period $\frac{K(k)}{\alpha}$; $K(k)$ is the complete elliptic integral of the first kind; sn are an elliptic sine, k

s the modulus of the elliptic functions, μ is restoring torque. Assumed that the problem can be decomposed into slowly and quickly changing variables, that one quickly changing variables (θ - the nutation angle) has periodicity, and thus that averaging to can be accomplished with a small error. We propose to carry out the investigation of the perturbed motion in the slow variables u_i $(i = 1, 2, 3)$, there are the real roots of cubic polynomial.

Slow variables u_i are connected via G_z, H and r by relations [1], which more convenient for analysis. Averaging the right hand sides of the resultant system over the phase of the nutation angle, we should obtain the averaged system of first approximation:

$$\frac{du_i}{d\tau} = U_i\left(u_1, u_2, u_3, \tau\right), u_i\left(0\right) = u_i^0, i = 1, 2, 3 \tag{1}$$

$$U_i\left(u_1, u_2, u_3, \tau\right) = \frac{\alpha}{2K\left(k\right)} \int_0^{\frac{2K}{\alpha}} V_i\left(u_1, u_2, u_3, \tau, \theta\left(t\right)\right) dt.$$

As an example,we investigate a perturbed motion, close to Lagrange's case, under the action of an external medium, that slowly changes the viscous properties.The averaged system is integrated numerically for various initial conditions and parameters of the problem. The graphs of the solutions were built. The dependence of the restoring and perturbation torques on the slow time leads to the exhibiting of the set of functions, which smooth out the behaviour of numerical dynamics of u_i $(i = 1, 2, 3)$, G_z and H at the stage of numerical integration. Under the influence of external dissipation, body tends to the stable state of lover equilibrium position more rapidly than in cases considered earlier [1, 2] (which follows from the specification of coefficients). New class of rotations of a dynamically symmetric rigid body about a fixed point has been investigated with restoring and perturbation unsteady torques being taken into account.

[1] Chernousko F. L., Akulenko L. D., Leshchenko D. D. *Evolution of Motions of a Rigid Body About its Center of Mass.* – Cham: Springer; 2017. – 241 p.

[2] Akulenko L. D., Zinkevich Ya. S., Kozachenko T. A., Leshchenko D. D. *The evolution of motions of a rigid body close to the Lagrange case under the action of an unsteady torque.* // Journal of Applied Mathematics and Mechanics. – 2017. – **82**, 2. – P. 79-84.